1965

# The effect of blow-down on the effective height of a helicopter supported VLF high-wire antenna

Coates, Dannie R.

Monterey, California. Naval Postgraduate School

# THE EFFECT OF BLOW-DOWN ON THE EFFECTIVE HEIGHT OF A HELICOPTER SUPPORTED VLF HIGH-WIRE ANTENNA

DANNIE R. COATES

54

# THE EFFECT OF BLOW-DOWN ON THE EFFECTIVE HEIGHT OF A HELICOPTER SUPPORTED VLF HIGH-WIRE ANTENNA

* * * * *

Dannie R. Coates

# THE EFFECT OF BLOW-DOWN ON THE EFFECTIVE HEIGHT OF A HELICOPTER SUPPORTED VLF HIGH-WIRE ANTENNA


by

Dannie R. Coates

Lieutenant, United States Navy


Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS


United States Naval Postgraduate School
Monterey, California

1965

# THE EFFECT OF BLOW-DOWN ON THE EFFECTIVE HEIGHT OF A HELICOPTER SUPPORTED VLF HIGH-WIRE ANTENNA

by

Dannie R. Coates

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

ENGINEERING ELECTRONICS

from the

United States Naval Postgraduate School

## ABSTRACT

The use of the very-low-frequency segment of the electromagnetic spectrum has been limited by the fact that practical VLF antennas are electrically short. It has been proposed that a helicopter be used to support a vertical radiator at heights of 7000 to 10,000 feet. This would give the antenna dimensions of the same order of magnitude as a quarter-wavelength. In general, the antenna will assume the shape of a catenary curve due to the wind forces acting on the cable. The equations that describe the antenna of general physical shape are presented and an analysis is given of the effects of the curvature on the effective height and radiation resistance of the antenna. Calculated data is presented to show how the antenna shape may be optimized in order to maximize the antennas effective height.

## TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS

## TABLE OF SYMBOLS AND ABBREVIATIONS

| | |
|---|---|
| LF | Low-frequency, 30 to 300 kilocycles |
| VLF | Very-low-frequency, 3 to 30 kilocycles |
| Q | Quality factor, $Q = X/R$ |
| $\lambda$ | Free space wavelength |
| $H_e$ | Effective height |
| H | Length of monopole |
| D | Diameter of monopole |
| $\theta$ | Angle of cable element from vertical |
| $Z_a$ | Antenna impedance |
| $R_a$ | Resistive component of antenna impedance |
| $X_a$ | Reactive component of antenna impedance |
| $R_r$ | Antenna radiation resistance |
| I | Antenna current |
| x,y | The rectangular coordinates of an arbitrarily chosen point on the cable |
| s | The distance along the cable measured positively in the sense of positive progression along the cable |
| $\phi$ | The angle from the direction of motion to the direction of the tangent to the cable at an arbitrarily chosen point on the cable |
| $\phi_o$ | The value of $\phi$ at the point chosen as the origin of the coordinate system |
| $\phi_c$ | The critical angle of the cable |
| F | The drag per unit length of the cable when the cable is parallel to the stream |
| R | The drag per unit length of the cable when the cable is normal to the stream |

$T$ The tension in the cable at an arbitrarily chosen point

$T_o$ The tension in the cable at the point chosen as origin of the coordinate system

$W$ The weight per unit length of the cable

$\tau$ The nondimensional tension, $T/T_o$

$\xi, \eta$ The nondimensional rectangular coordinates;

$$\xi = \frac{Rx}{T_o} \; ; \; \eta = \frac{Ry}{T_o}$$

$\sigma$ The nondimensional length of cable, $Rs/T_o$

$f$ The ratio $F/R$

$w$ The ratio $W/R$

1. Introduction

Although recent development of the use of the electro-magnetic spectrum has occured mainly at medium and higher frequencies, the propagation characteristics of low-frequency (LF) and very-low-frequencies (VLF) waves still tend to recommend that region of the electro-magnetic spectrum for certain special services. In these frequency ranges, ground waves are subject to far less attenuation and sky waves are much less affected by ionospheric distrubances. These characteristics make the region especially favorable for long-range communications and for navigational aids to widely dispersed ships and aircraft. Another characteristic, applying especially to VLF, is the ability of the electro-magnetic energy to penetrate into salt water, making communications with submerged submarines possible.

Three of the major considerations in very-low-frequency antenna design are the amount of power to be radiated in order to obtain a required area of communications coverage, the efficiency of the antenna, and obtaining a low value of antenna Q, in order to satisfy a bandwidth requirement of the signal to be transmitted. In many ways these performance requirements are similar when a simple vertical antenna is used. The quality of a vertical antenna with regard to efficiency, power capacity, and bandwidth increase as the electrical length of the antenna is increased to values approximating a quarter wavelength.

From figure 1-1 can be seen that in the VLF range the wavelength is very long. Therefore all practical fixed VLF antennas will be electrically short. Figure 1-2 shows the radiation resistance as a function of electrical length. For the example of a vertical antenna consisting of a one-thousand foot tower operating at a frequency of 20 kilocycles the electrical length will be approximately $0.02\lambda$ and the radiation resistance is 0.18 ohms. This low value of radiation resistance is of the same order of magnitude as the electrical loss resistance of the antenna. Therefore the power loss will be of the same order of magnitude as the power radiated and so the efficiency of the electrically short antanna will be low. Because the radiation resistance is so low for antennas of practical length, large values of antenna current must be used in order to radiate an appreciable amount of power. Figure 1-2 also shows that there is a large reactive component associated with the impedance of the short antenna. The combination of large antenna current and large reactance will induce large voltages, $(V = I_a X_a)$, on the antenna. If this voltage is driven too high corona discharge will take place. Therefore the power handling capacity of the antenna is limited to values below which corona becomes a problem. Also, the combination of large reactance and small resistance gives a large value of antenna Q. This large value of antenna

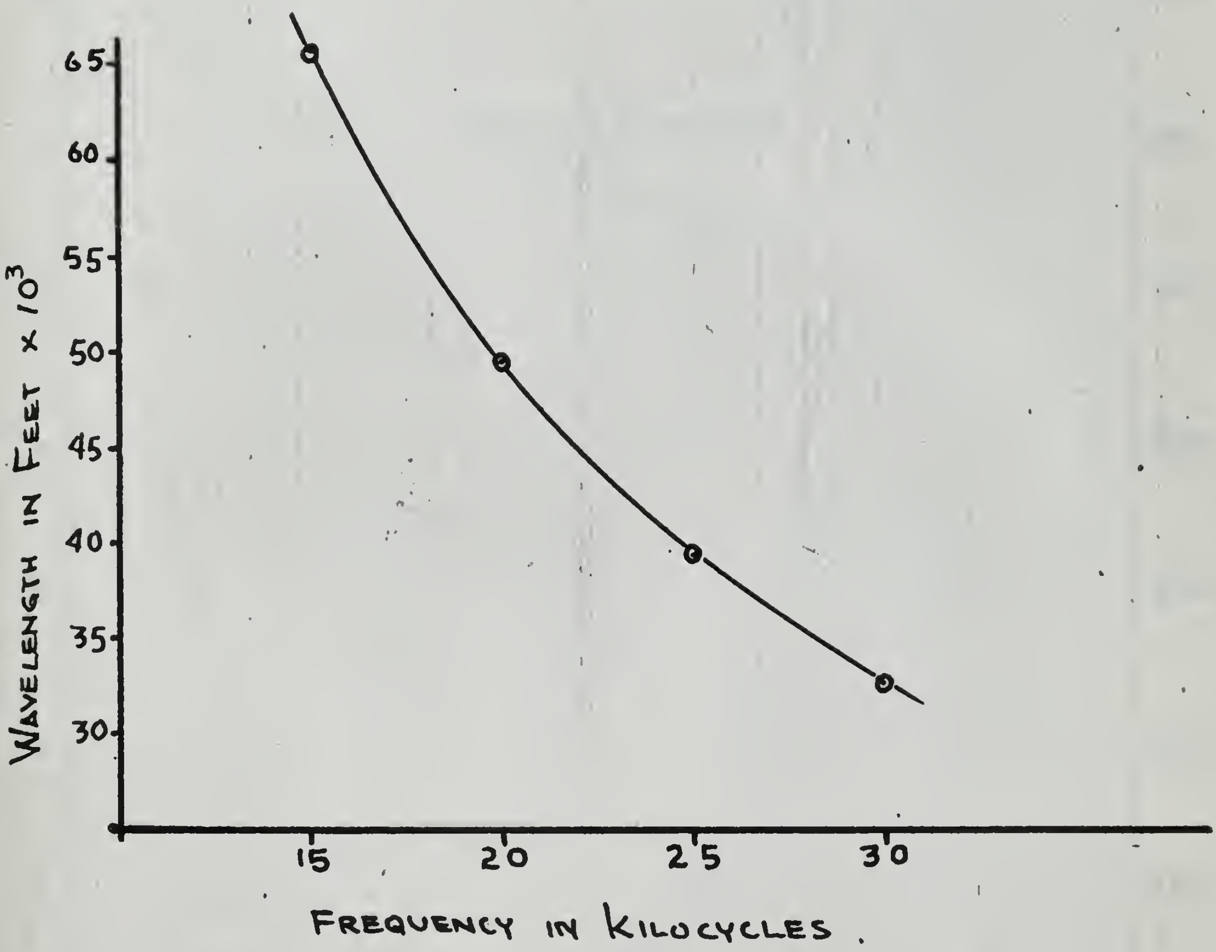



FIGURE 1-1. WAVELENGTH AS A FUNCTION OF FREQUENCY

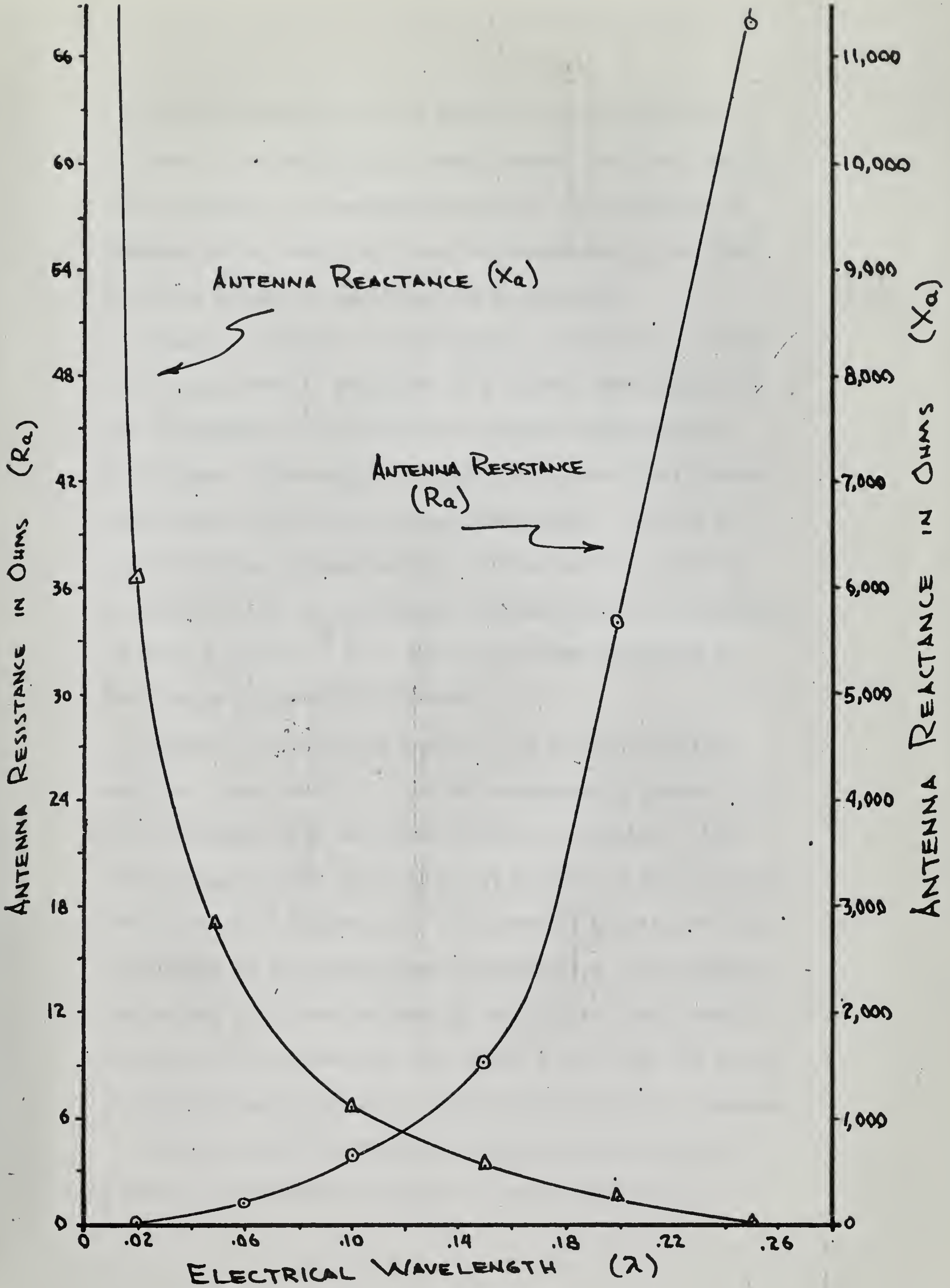



FIGURE 1-2. ANTENNA IMPEDANCE AS A FUNCTION OF ELECTRICAL WAVELENGTH

Q limits the bandwidth of the signals to be transmitted. For most of the existing VLF installations the signal modulation is limited to slow-speed CW keying. The bandwidth of frequency shift keying and voice transmissions is too great for these methods of modulation to be utilized.

Ideally, a simple vertical rediator should have a length of the same order of magnitude as a quarter wavelength, but for the example of 20 kilocycles a quarter wavelength is 12,300 feet. This height is approximately ten times greater than modern, rigid tower construction allows. The use of a high-wire antenna supported by a lifting airframe, such as a tethered ballon or an unmanned remore-controlled helicopter, has been suggested [6] as a rapid and economical method of erecting an efficient VLF radiator.

Ideally, the high-wire antenna is a vertical radiator anywhere from a tenth to a quarter wavelength in height. The wire in general will be curved rather than vertical. The wire is supported by the aircraft at heights of 7000 to 10,000 feet as shown in figure 1-3. The curvature of the wire will be governed by wind conditions along the wire. The displacement of the wire from vertical by the wind has been termed blow-down. The purpose of this report is to study the effect of blow-down on the effective height of the high-wire antenna.

The term effective height is defined as the length of a vertical wire which, if caused to carry a uniformly

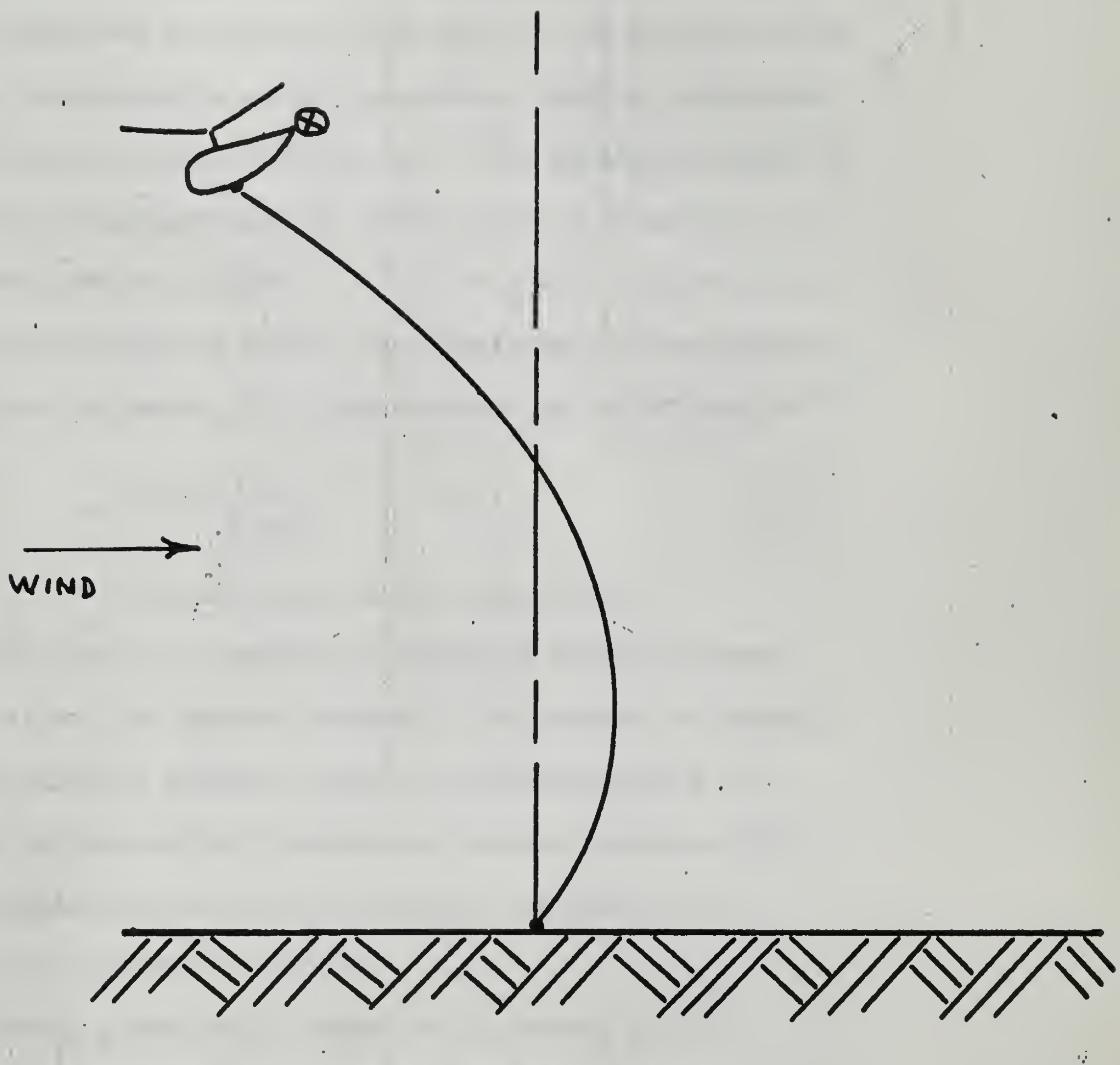



FIGURE 1-3. HIGH-WIRE ANTENNA OF GENERAL CONFIGURATION

distributed current equal to the input current of the antenna, would produce a distant field strength on the horizon equal to that of the antenna, assuming infinite ground conductivity in both cases [2]. The high-wire antenna under consideration consists of a vertical radiator fed against ground. Under these conditions the "image" forms half of the antenna system. In this terminology a vertical antenna a quarter wavelength in height has a half-length of $\lambda/4$. The effective height of a quarter wavelength vertical antenna with a sinusoidal current distribution is $(2/\pi) \times (\lambda/4) = \lambda/2\pi$. The radiation resistance of such an antenna is proportional to the square of the effective height, $(h_e)$, and is given by the expression [2]

$$R_{ar} = 160\pi^2 \left(\frac{h_e}{\lambda}\right)^2 \quad \text{ohms} \tag{1-1}$$

where $h_e$ and $\lambda$ are expressed in the same units.

The effect of blow-down on effective height is threefold. First, the physical height of the antenna is reduced. The reduction in physical height is accompanied by a reduction in the vertical component of antenna current. Both the effective height and the radiation resistance of the antanna are therefore reduced.

Second, a horizontal component of antenna current is introduced. Radiation by this horizontal component of current is suppressed by the image currents in the ground plane over which the vertical monopole is operated. The image currents

flow in a direction opposite to the antenna currents. When the separation between these two currents is small, their radiation fields cancel. As the separation increases, the cancellation becomes partial and radiation occurs. Since the separation is proportional to antenna length, the effect is more noticeable for the larger antennas. A quarter wavelength antenna has a sufficient separation between the antenna current and its image to produce significant radiation by the horizontal component of current. When the horizontal component of current radiates, the effect is to decrease the radiation resistance during blow-down less rapidly than if just the vertical component of the current radiated.

The third effect of blow-down is to change, in magnitude, the distribution of current along the antenna. The current decreases more rapidly away from the feed point after blow-down than before. The result is a decrease in effective height or equivalently in radiation resistance.

In the following analysis, the first two effects of blow-down on radiation resistance are studied; viz., the reduction in effective height and the introduction of horizontally polarized radiation. The third effect, the change of current distribution along the wire was not studied at this time. The first two effects were studied for both a tilted monopole and for a curved wire. The purpose of studying the tilted monopole is to check the method of analysis against a known solution and to provide greater

physical insight into the method of analysis. This method of analysis is then applied to a curved wire which better represents the actual geometry of the high-wire antenna when subjected to blow-down.

By assuming a given set of cable parameters the above analysis will be used to generate data showing the optimum maneuver a lifting helicopter should fly in order to maximize the effective height of the antenna for various wind conditions.

2. General Discussion

Section 4 of this report gives a detailed description of the method used to evaluate the effective height of the curved wire antenna of general shape. Briefly, the effective height calculations require that the local angle of tilt of the antenna from vertical be known as a function of the distance along the antenna as measured from the base of the antenna. It is also required that the antenna current be known as a function of this same distance. For all of the effective height calculations the antenna current is assumed to have a sinusoidal distribution, i.e., the antenna current will be a maximum at the base of the antenna and will diminish, in proportion to sin $(\beta\ell)$, to zero at the top.

Section 3 describes the method used to determine the shape of the antenna as a function of wind velocity. This analysis results in a set of four integral equations that

are called cable functions. These cable functions describe the tension in the cable at each point, the distance of each point from a reference point, and the x and y coordinate of each point all as functions of the local tilt angle of that point.

In order to reduce the number of parameters in the effective height calculations several assumptions will be made. The wind speed will be assumed constant along the length of the cable in order to greatly simplify the mathematics of calculating the cable curves. The wind direction will be assumed to be constant along the cable from ground level to the top so that the cable curve will lie in a single plane. It will be assumed that the cable is a smooth cylinder and that it is perfectly flexible with the physical characteristics described in Section 5a. Physical cable lengths of 7000 to 10,000 feet and values of cable tension at the top of the antenna of 4000 to 10,000 pounds will be considered. The values of antenna electrical length will be varied from one-eighth to one-quarter wavelength. Values of cable angle at the top will be varied from the critical angle to the vertical. The critical angle is defined in Section 3.

Section 6 presents curves showing how effective height varies with the parameters listed above, i.e., wind speed, tension at the top of the cable, and cable angle at the top.

3. Cable Function Calculations

A definitive treatment of the problem of defining the configuration of a tethering cable in a uniform wind is given by Pode [5]. Briefly, the treatment given is as follows. The fact that the normal force acting on a round cylinder in a uniform stream varies as the square of the sine of the angle between the cylinder and the stream has been thoroughly established by experiment and supported by theoretical studies. If it is assumed that the cable is perfectly flexible and that the aerodynamic forces acting on an element of the cable depends only on the angle that the element makes with the stream and are not affected by such factors as curvature of the cable or flow at neighboring elements, differential equations can be written which describe, in non-dimensional coordinates, the equilibrium configuration of the cable in a uniform stream. In cases where the gravitational forces acting on an element of cable cannot be neglected in comparison with the aerodynamic forces, the equations are not integrable directly but numerical methods must be used.

If both the direction of gravity and the law relating the aerodynamic force to the angle between an element of cable and the stream are not specified, the external force acting upon an element of cable is a known function of this angle [5]. The components of the force parallel to the element of the cable and normal to the element of the cable may both be

written as explicit functions of this angle.

Choose a sense of progression along the cable and let $\phi$ be the angle measured counterclockwise from the direction of motion to the direction of an element of cable of length, ds. Let P $(\phi)$ds and Q $(\phi)$ds be the tangential and normal components of the external forces respectively, where P $(\phi)$ is measured positive in the direction of the element of cable which is taken in the sense of increasing length of cable, s, in accordance with the chosen sense of progression, and Q $(\phi)$ is measured positive in the direction of the positive normal which is taken in the direction 90 degrees counterclockwise from the direction of the element of the cable. Then the equilibrium of the cable element requires [5]:

$$dT = -P\ (\phi) ds \qquad (3\text{-}1)$$

$$Td = -Q\ (\phi) ds \qquad (3\text{-}2)$$

where T is the tension in the cable and dT and d$\phi$ are the changes in the values of T and $\phi$ over the length of the element; see figure 3-1. Since the forces that act on an element of the cable cannot be affected by the choice of the sence of progression along the cable, the functions P $(\phi)$ and Q $(\phi)$ must satisfy the relations:

$$P\ (\phi) = -P\ (\phi + \pi)$$

$$Q\ (\phi) = -Q\ (\phi + \pi)$$

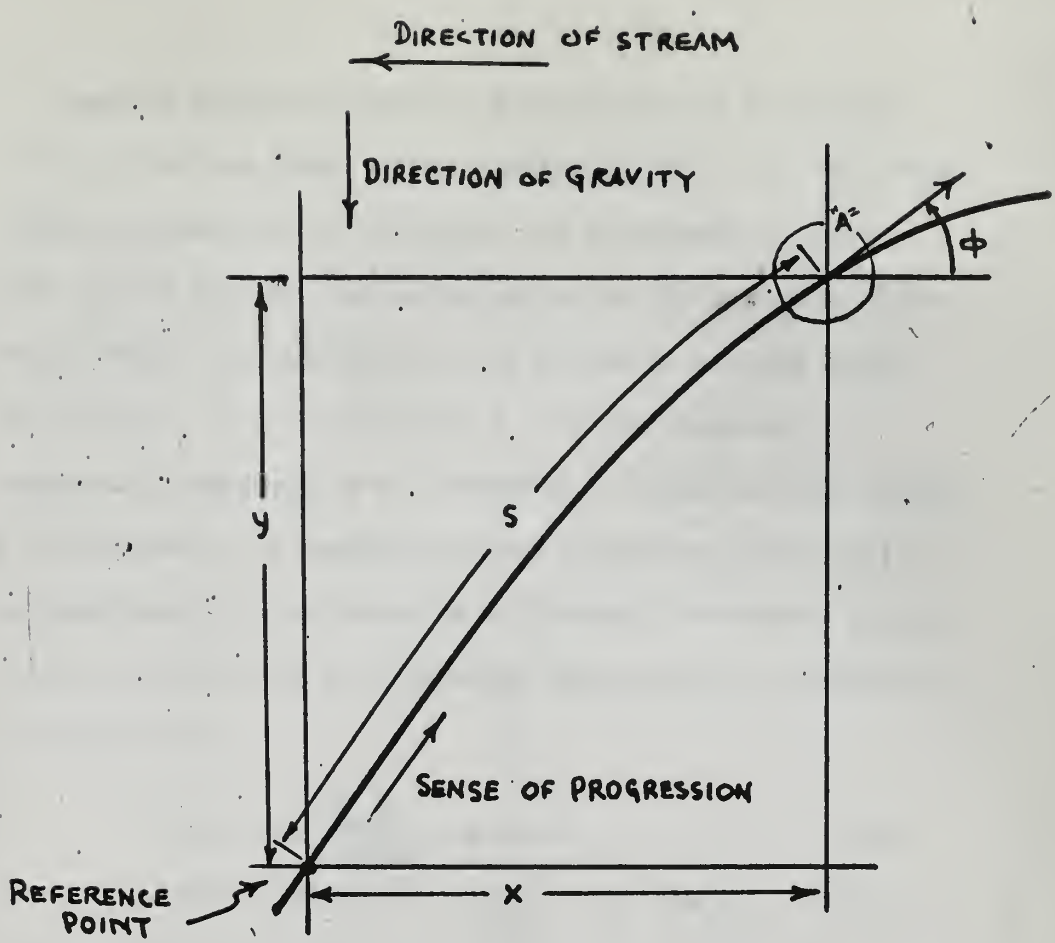



FIGURE 3-1a. COORDINATE SYSTEM

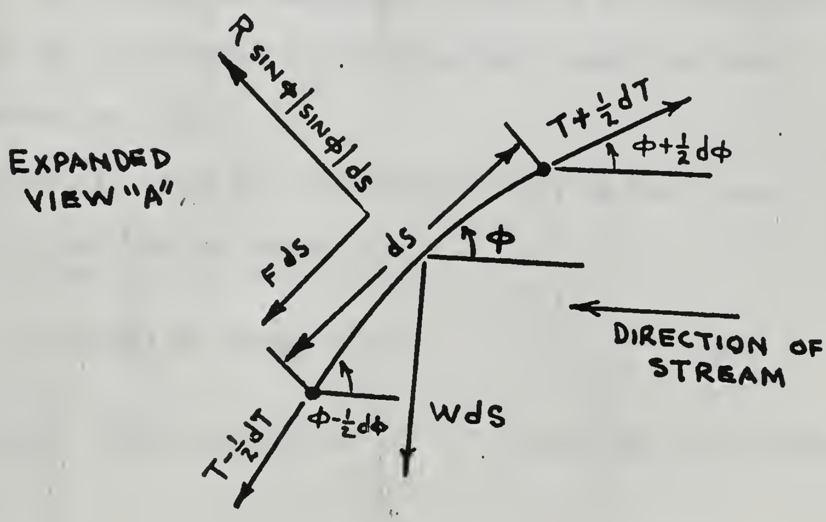



FIGURE 3-1b. FORCES ACTING ON AN ELEMENT OF CABLE

Special interest attaches to the values of the angle $\phi = \phi_c$ which are roots of the equation $Q(\phi) = 0$. When the cable anchored only at the upper end is allowed to hang free in the stream, the configuration of the cable could be any straight line inclined to the stream at such an angle $\phi = \phi_c$. If, for simplicity, the magnitude of the tangential component of the aerodynamic force per unit length F is assumed to be constant and the direction of motion is perpendicular to the direction of gravity, the forces to be considered acting on an element of cable are as represented in figure 3-1b.

$$P(\phi) = -F \frac{\cos\phi}{|\cos\phi|} - W \sin\phi \qquad (3\text{-}3)$$

$$Q(\phi) = R \sin\phi \, |\sin\phi| - W \cos\phi \qquad (3\text{-}4)$$

where W is the weight per unit length of the cable. The sign of $F \frac{\cos\phi}{|\cos\phi|}$ is proper in order to take into account the fact that the tangential component as well as the normal component of the aerodynamic force never has a positive projection into the wind.

The critical angle may be assumed to lie in the range $0 \leq \phi_c \leq \pi$ so that the equation

$$R \sin^2\phi_c - W \cos\phi_c = 0 \qquad (3\text{-}5)$$

is satisfied. Substituting $\sin^2\phi_c = 1 - \cos^2\phi_c$ and dividing

by R

$$\cos^2 \phi_c + \frac{W}{R} \cos \phi_c - 1 = 0 \qquad (3\text{-}6)$$

Hence

$$\cos \phi_c = \frac{-W}{2R} + \sqrt{\left(\frac{W}{2R}\right)^2 + 1} \qquad (3\text{-}7)$$

The general intergration of equations (3-1) and (3-2) may now proceed. Eliminating ds from equations (3-1) and (3-2)

$$\frac{dT}{T} = \frac{P(\phi)}{Q(\phi)} d\phi \qquad (3\text{-}8)$$

Now assume that at some point, $P_o$, on the cable, the tension in the cable, $T_o$, and the angle from the direction of motion, $\phi_o$, are known. Equation (3-8) may be integrated from this reference point, $P_o$, along the cable to any arbitrary point, P, on the cable where the tension if T and the angle is $\phi$:

thus

$$\frac{T}{T_o} = \exp\left[\int_{\phi_o}^{\phi} \frac{P(\phi)}{Q(\phi)} d\phi\right] \qquad (3\text{-}9)$$

Using this result in equation (3- 2)

$$ds = \frac{T_o}{-Q(\phi)} \exp\left[\int_{\phi_o}^{\phi} \frac{P(\phi)}{Q(\phi)} d\phi\right] \qquad (3\text{-}10)$$

so that the distance along the cable from $P_o$ to P is given by

$$s = \int_{\phi_o}^{\phi} \frac{T_o}{-Q(\phi)} \exp\left[\int_{\phi_o}^{\phi} \frac{P(\phi)}{Q(\phi)} d\phi\right] d\phi \qquad (3\text{-}11)$$

The location of the point P in relation to the point $P_o$ may be found in terms of coordinates x and y, representing a displacement parallel to the direction of motion and displacement perpendicular to the direction of motion respectively. From the geometry of the configuration,

$dx, = (\cos \phi)\ ds$

and $dy = (\sin\phi)ds$; hence

$$x = \int_{\phi_0}^{\phi} \frac{T_0}{-Q(\phi)} \exp\left[\int_{\phi_0}^{\phi} \frac{P(\phi)}{Q(\phi)} d\phi\right] \cos\phi \, d\phi \quad (3\text{-}12)$$

$$y = \int_{\phi_0}^{\phi} \frac{T_0}{-Q(\phi)} \exp\left[\int_{\phi_0}^{\phi} \frac{P(\phi)}{Q(\phi)} d\phi\right] \sin\phi \, d\phi \quad (3\text{-}13)$$

In order to obtain solutions that are applicable to a cable configuration of most general shape, these solutions may be reduced to a non-dimensional form. The tension T is already in non-dimensional form in terms of the known tension $T_0$. For the distances s, x, and y, a characteristic unit of length is needed. In general, the most convenient unit of length is that length of cable which when entirely normal to the stream has a drag equal to the tension $T_0$, i.e., $\frac{T_0}{R}$ where R is the drag per unit length when the cable is normal to the stream. Dividing the distances s, x, and y by this length the non-dimensional values $\sigma = \frac{Rs}{T_0}$, $\xi = \frac{Rx}{T_0}$, $\eta = \frac{Ry}{T_0}$, are obtained. Then letting $p = \frac{P(\phi)}{R}$, $q = \frac{Q(\phi)}{R}$ and using equations (3-9), (3-11), (3-12), and (3-13) the solution of the cable problem may be written:

$$\tau = \frac{T}{T_0} = \exp\left[\int_{\phi_0}^{\phi} \frac{p}{q} d\phi\right] \quad (3\text{-}14a)$$

$$\sigma = \frac{Rs}{T_0} = \int_{\phi_0}^{\phi} \frac{\tau}{-q} d\phi \quad (3\text{-}14b)$$

$$\xi = \frac{Rx}{T_0} = \int_{\phi_0}^{\phi} \frac{\tau\cos\phi}{-q} d\phi \quad (3\text{-}14c)$$

$$\eta = \frac{Ry}{T_0} = \int_{\phi_0}^{\phi} \frac{\tau\sin\phi}{-q} d\phi \quad (3\text{-}14d)$$

where only non-dimensional values are involved and all functions are defined by quadratures.

Equations (3-3) and (3-4) may now be substituted into equations (3-14a, b, c, d) and so the cable functions may be written

$$\ln \tau = \int_{\phi_0}^{\phi} \frac{f \dfrac{\cos\phi}{|\cos\phi|} + w \sin\phi}{-\sin\phi\,|\sin\phi| + w \cos\phi}\, d\phi \qquad (3\text{-}15a)$$

$$\sigma = \int_{\phi_0}^{\phi} \frac{\tau}{-\sin\phi\,|\sin\phi| + w \cos\phi}\, d\phi \qquad (3\text{-}15b)$$

$$\xi = \int_{\phi_0}^{\phi} \frac{\tau \cos\phi}{-\sin\phi\,|\sin\phi| + w \cos\phi}\, d\phi \qquad (3\text{-}15c)$$

$$\eta = \int_{\phi_0}^{\phi} \frac{\tau \sin\phi}{-\sin\phi\,|\sin\phi| + w \cos\phi}\, d\phi \qquad (3\text{-}15d)$$

where $f = F/R$ and $w = W/R$. For calculating the cable functions it is convenient to divide the integrations into the three quadrants, see figure (3-2), in which the angle $\phi$ may fall, namely:

Quadrant 1 where $\phi_c \leq \phi \leq \pi/2$

Quadrant 2 where $\pi/2 \leq \phi \leq \pi$

Quadrant 3 where $\pi \leq \phi \leq \pi + \phi_c$

The configuration of a cable used to tether an aircraft is such that only quadrants 1 and 2 need be considered. For all calculations to be made in this report the point $\phi = \pi/2$ will be used as the reference point and the following cable function equations will be used:

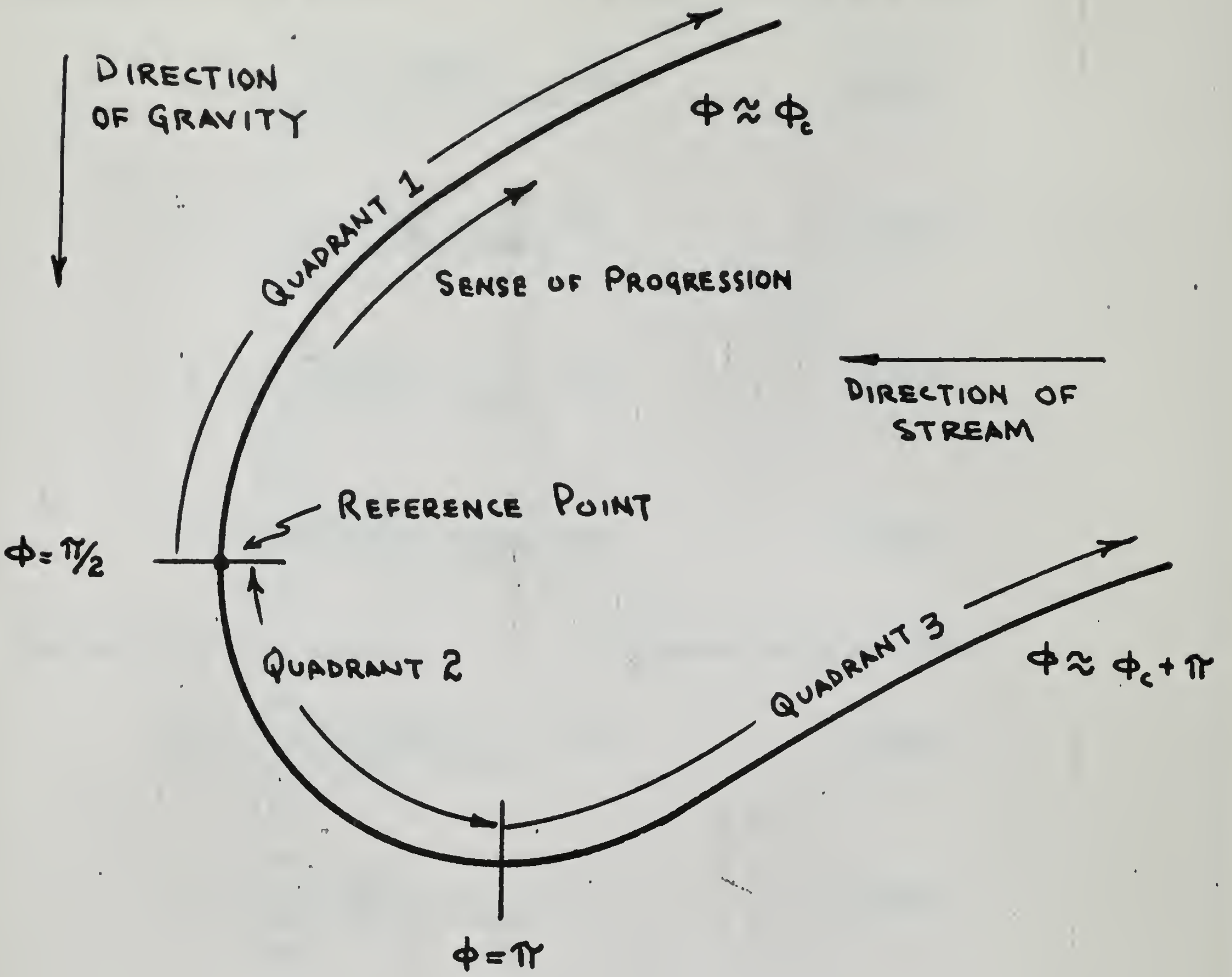



FIGURE 3-2. GENERAL CONFIGURATION OF A HEAVY CABLE IN A UNIFORM STREAM

Quadrant 1 $\equiv \phi_c < \phi \leq \pi/2$ Reference point $\phi = \pi/2$

$$\ln \tau = \int_{\phi_0 = \pi/2}^{\phi} \frac{f + w \sin\phi}{-\sin^2\phi + w \cos\phi} d\phi \tag{3-16a}$$

$$\sigma = \int_{\pi/2}^{\phi} \frac{\tau}{-\sin^2\phi + w \cos\phi} d\phi \tag{3-16b}$$

$$\xi = \int_{\pi/2}^{\phi} \frac{\tau \cos\phi}{-\sin^2\phi + w \cos\phi} d\phi \tag{3-16c}$$

$$\eta = \int_{\pi/2}^{\phi} \frac{\tau \sin\phi}{-\sin^2\phi + w \cos\phi} d\phi \tag{3-16d}$$

Quadrant 2 $\equiv \pi/2 \leq \phi \leq \pi$ Reference point $\phi = \pi/2$

$$\ln \tau = \int_{\pi/2}^{\phi} \frac{-f + w \sin\phi}{-\sin^2\phi + w \cos\phi} d\phi \tag{3-17a}$$

$$\sigma = \int_{\pi/2}^{\phi} \frac{\tau}{-\sin^2\phi + w \cos\phi} d\phi \tag{3-17b}$$

$$\xi = \int_{\pi/2}^{\phi} \frac{\tau \cos}{-\sin^2\phi + w \cos\phi} d\phi \tag{3-17c}$$

$$\eta = \int_{\pi/2}^{\phi} \frac{\tau \sin\phi}{-\sin^2\phi + w \cos\phi} d\phi \tag{3-17d}$$

4. Analysis of Tilted Monopole

a. Composite Solution

In this analysis, the antenna during blow-down is simply represented as a tilted monopole over a ground plane. The

impedance of the tilted monopole is just one-half the impedance of an associated V-antenna formed by the tilted monopole and its image. Schelkunoff[8] gives the impedance of the V-antenna as

$$Z_v = K_a \frac{(K_a - M) \cos\beta H + j\,(Z_a - jN) \sin\beta H}{(Z_a + jN) \cos\beta H + j\,(K_a + M) \sin\beta H} \qquad (4\text{-}1)$$

where

$$M = 60\,(\mathrm{Cin}\,2\beta H - 1 + \cos 2\beta H)$$

$$N = 60\,(\mathrm{Si}2\beta H - \sin 2\beta H)$$

$$K_a = 120\left[\ln\left(\frac{4H}{D}\right) - 1 + \ln(k)\right]$$

$$\mathrm{Cin}\,(x) = \int_0^x \frac{1 - \cos y}{y}\,dy$$

$$\mathrm{Si}(x) = \int_0^x \frac{\sin y}{y}\,dy$$

$$Z_a = R_a + jX_a$$

$$\begin{aligned} R_a = {} & 60\,\mathrm{Cin}2\beta Hk \\ & +30\left[2\mathrm{Cin}2\beta H - \mathrm{Cin}2\beta H(1-k) - \mathrm{Cin}2\beta H(1+k)\right]\cos 2\beta H \\ & +30\left[-2\mathrm{Si}2\beta H + \mathrm{Si}2\beta H(1-k) + \mathrm{Si}2\beta H(1+k)\right]\sin 2\beta H \end{aligned}$$

$$\begin{aligned} X_a = {} & 60\,\mathrm{Si}2\beta Hk \\ & +30\left[\mathrm{Si}2\beta H(1-k) - \mathrm{Si}2\beta H(1+k)\right]\cos 2\beta H \\ & +30\left[\mathrm{Cin}2\beta H(1-k) - \mathrm{Cin}2\beta H(1+k) + 2\ln(1+k)\right]\sin 2\beta H \end{aligned}$$

$H$ = length of monopole

$D$ = diameter of monopole

$k = \cos\Theta$

$\theta$ = angle off vertical

$\beta$ = $2\pi/\lambda$

$\lambda$ = free space wavelength

Two simplifications of the expression for radiation resistance of the V-antenna are possible. The first simplification is for electrically small antennas. The second simplification is for antennas near self-resonance.

For electrically small antennas ($H/\lambda \ll 1$), the following approximations can be made:

$$R_a \approx 20k^2 \, (\beta H)^4 \qquad (4\text{-}2)$$

$$X_a \approx 120\,\beta H \;\; \ln(1+k)$$

$$M \approx -60(\beta H)^2$$

$$N \approx 400/9 \;\; (\beta H)^3$$

Using these values, the radiation resistance of the V-antenna can be approximated by the expression

$$R_r \approx \left[ \frac{\ln\left(\frac{4Hk}{D}\right) - 1}{\ln\left(\frac{4Hk}{D}\right) + \ln(1+k) - 1} \right]^2 \; 20\,(\beta Hk)^2 \qquad (4\text{-}3)$$

For large height-to-diameter rations and reasonable tilt-angles [3] ($2Hk/D \gg 1$),

$$R_r = 20\,(\beta Hk)^2 \qquad (4\text{-}4)$$

As the monopole tilts off the vertical by an angle $\theta$, the radiation resistance decreases by the factor

$$\frac{R_r}{R_{ro}} = \cos^2\theta \qquad (4\text{-}5)$$

The subscript zero refers to the vertical antenna ($\theta = 0$). The radiation resistance of the short monopole is plotted in figure 4-1 as a function of tilt angle. Since the effective height of the antenna is proportional to the square root of its radiation resistance, the effective height of the short monopole decreases by the same factor as does its projected height.

$$\frac{H_e}{H_{eo}} = \cos \theta \tag{4-6}$$

A second simplification in the expression for radiation resistance is possible for the case of a tilted monopole near resonance ($\beta H - \pi/2 \ll 1$). Near resonance, the expression for the radiation resistance can be approximated by the expression

$$R_r \approx \frac{K_a}{K_a + M} R_a \tag{4-7}$$

The reduction in radiation resistance produced by tilting the near resonance monopole is given by the expression

$$\frac{R_r}{R_{ro}} = \frac{K_a(K_{ao} + M)\ R_a}{K_{ao}\ (K_a + M)\ R_{ao}} \tag{4-8}$$

For large height-to-diameter rations and reasonable tilt angles, the following approximations can be used

$$K_a\ (K_{ao} + M) \approx K_{ao}\ (K_a + M) \tag{4-9}$$

For this case, the reduction in radiation resistance is given by the expression

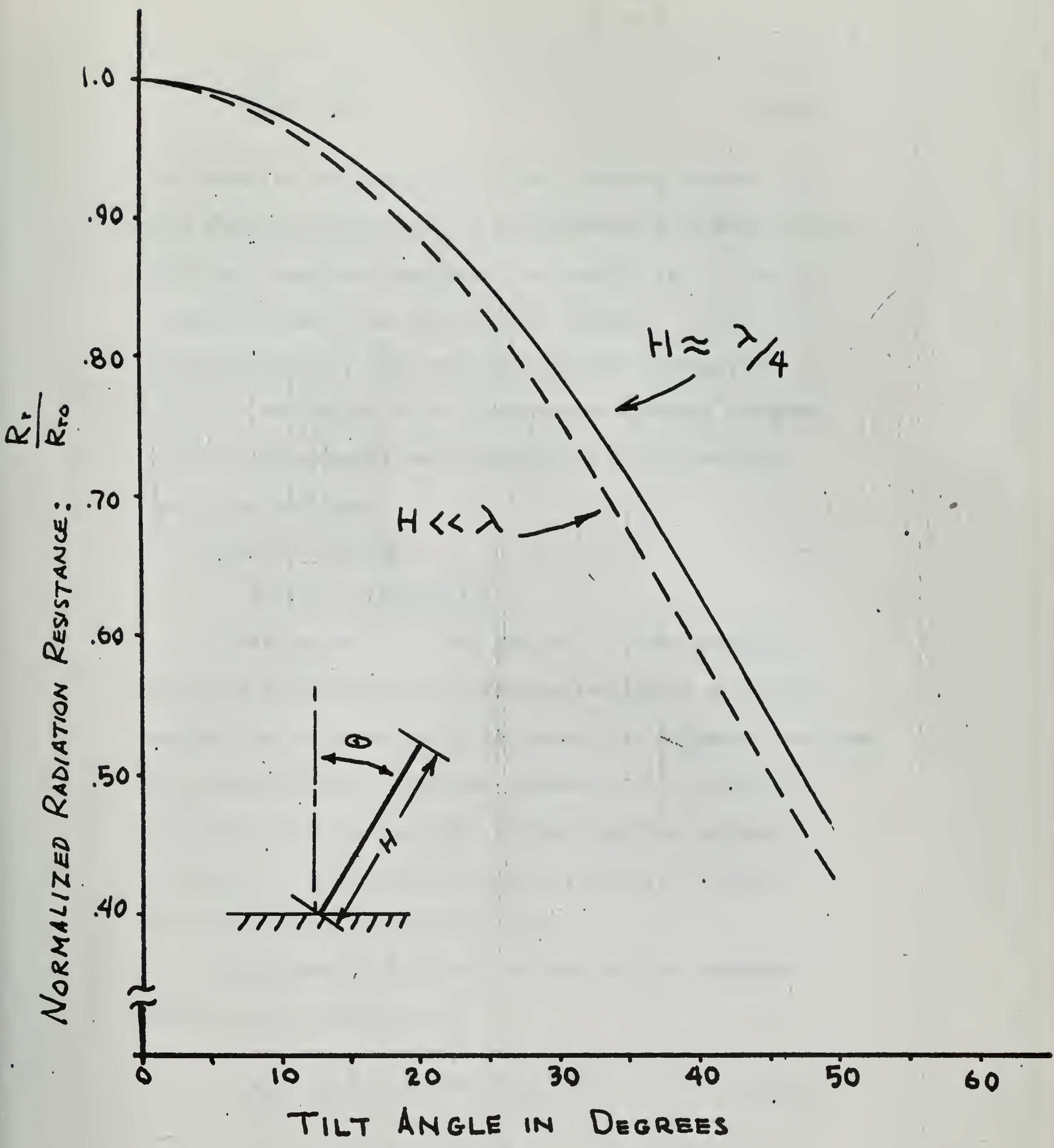
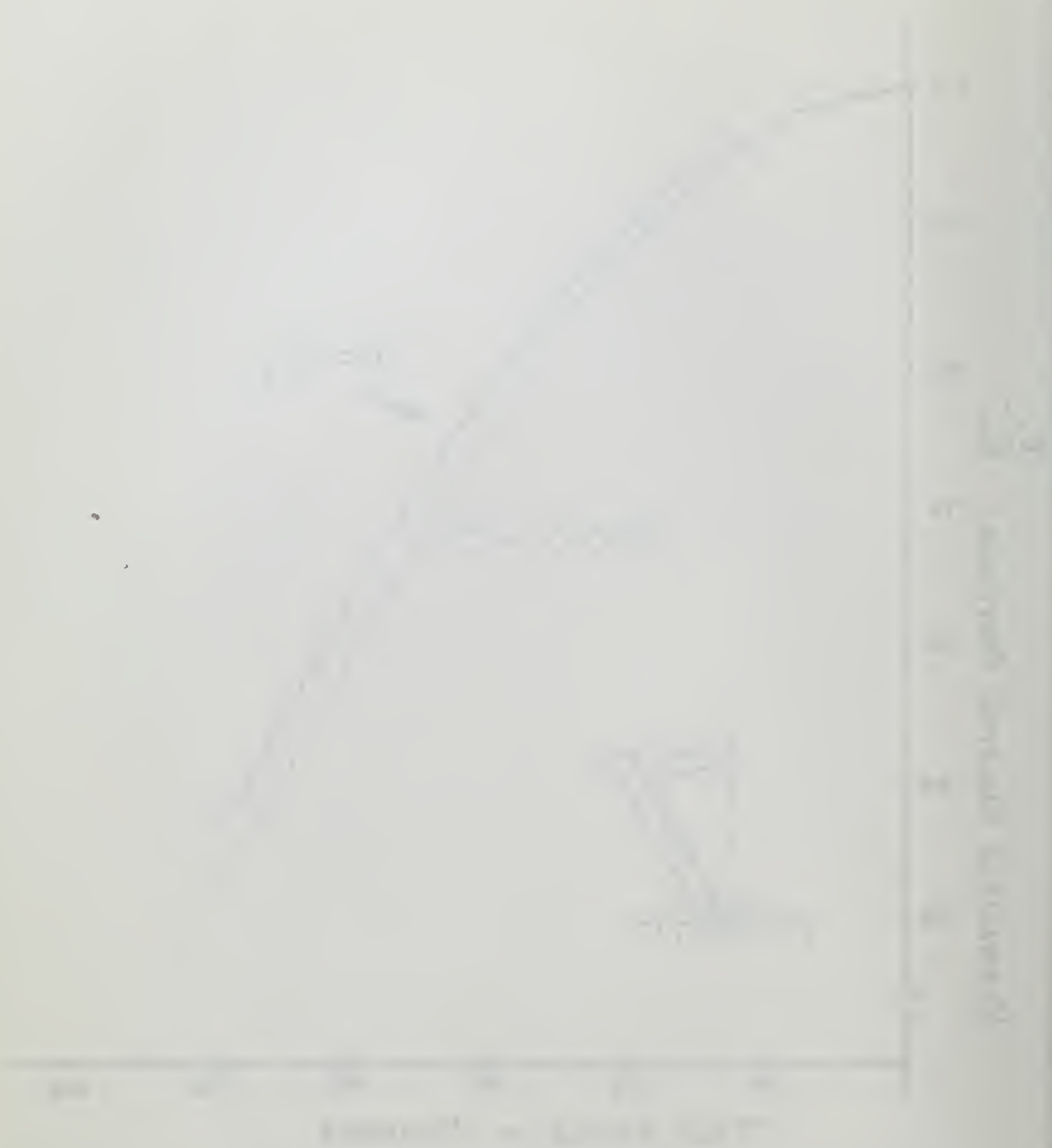



Figure 4-1. Reduction in Radiation Resistance for a Tilted Monopole



$$\frac{R_r}{R_{ro}} \approx \frac{R_a}{R_{ao}} \qquad (4\text{-}10)$$

The radiation resistance of the near resonant monopole is also plotted in figure 4-1 as a function of tilt angle. The radiation resistance decreases less rapidly for the near resonant antenna than for the short antenna. It will be shown subsequently that this more moderate decrease in radiation resistance of the quarter-wave monopole compared to the short monopole can be attributed to horizontally polarized radiation.

b. Component Solution

1. Vertical Polarization

It will be shown in this section that the preceding composite solution for the radiation resistance of a tilted monopole can be generated by the sum of two component solutions, one representing the radiation resistance of a vertically polarized field and the other representing the radiation resistance of a horizontally polarized field. Consider first the vertically polarized field.

The expression which will be used for the effective height of the antenna is [4]

$$H_e = \frac{1}{I_o}\int_0^H I(s) \cos\left[\Theta(s)\right] ds \qquad (4\text{-}11)$$

where

H = length of antenna

s = distance along antenna measured from the base

$\theta$ = local angle of tilt off the vertical

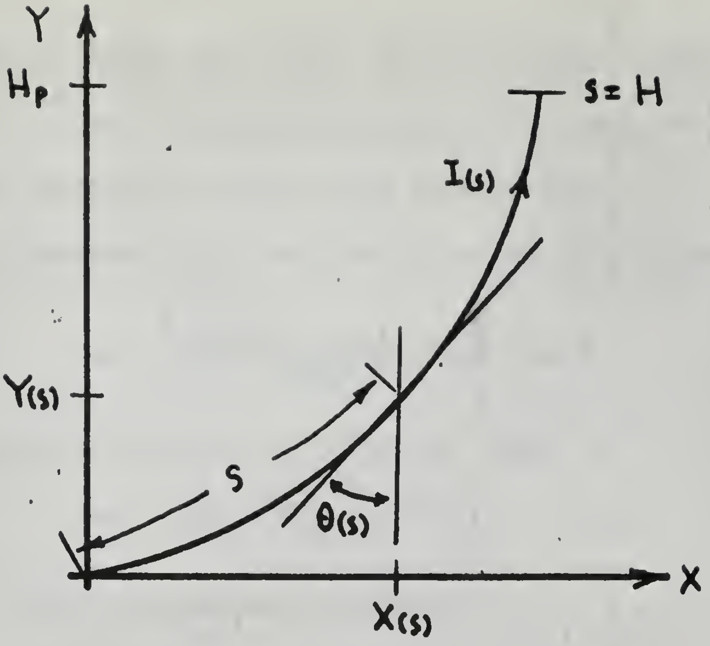
I = antenna current

The expression applies to the general curved antenna such as sketched in figure 4-2a. Only the vertical component of the current element is involved in equation (4-11) and so it accounts only for the vertically polarized radiation. A physical interpretation of this equation is suggested in figure 4-2b. It is assumed that the radiation from the stepped radiation structure is the same as for the smooth structure as the size of the steps approaches zero. The radiating fields from all the vertical elements are summed. Any radiation from the horizontal elements has been ignored but will be accounted for later. Phase differences between the radiated fields are ignored as are pattern differences between the various element-image pairs. In order to justify these approximations, the method will be applied to the tiltied monopole and the solution compared to the more exact solution cited previously.

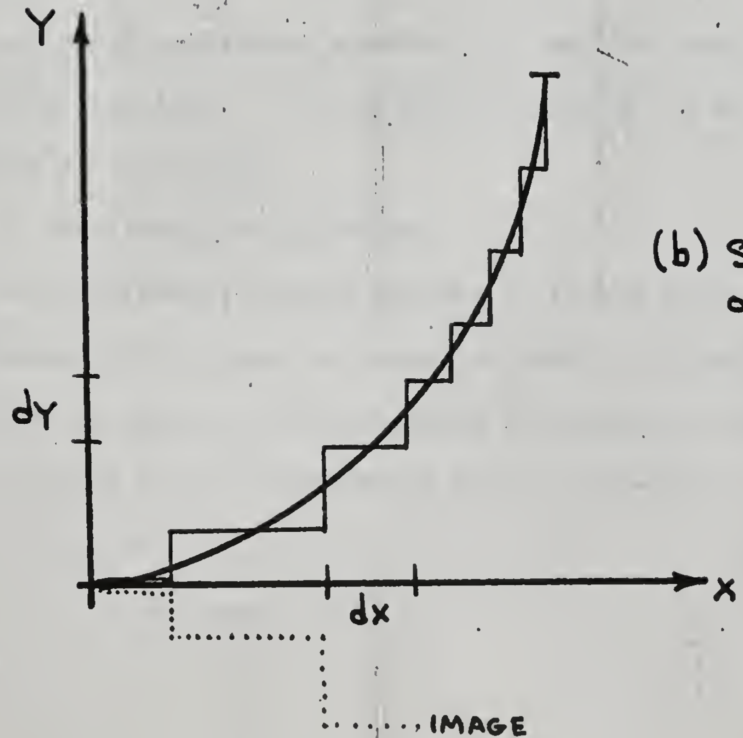
For the tilted monopole, $\theta$ is constant. A sinusoidal current will be assumed

$$I_{(s)} = I_o \frac{\sin\beta (H - s)}{\sin\beta H} \qquad (4\text{-}12)$$

The choice of current representation is not as critical when relative values of radiation resistance are needed as when




(a) Smooth Curve




(b) Stepped Approximation of Smooth Curve

Figure 4-2. Geometry of Curved Wire Antenna

absolute values are needed. The sinusoidal current will be used for the tilted monopole, just as it will be used in later examples for the curved wire antennas.

Equation (4-11) for the tilted monopole becomes

$$H_e = \left[ \int_0^H \frac{\sin \beta (H - s) \quad ds}{\sin \beta H} \right] \cos \theta \qquad (4\text{-}13)$$

At zero tilt angle, the effective height is

$$H_{eo} = \frac{2}{\beta} \left( \frac{\sin^2 \beta H/2}{\sin \beta H} \right) \qquad (4\text{-}14)$$

The relative radiation resistance is

$$\frac{H_e}{H_{eo}} = \cos \theta \qquad (4\text{-}15)$$

The relative radiation resistance is

$$\left( \frac{R_r}{R_{ro}} \right)_v = \cos^2 \theta \qquad (4\text{-}16)$$

This radiation resistance accounts for only the vertically polarized radiation. It is plotted in figure 4-3 as a function of tilt angle.

2. Horizontal Polarization

Each horizontal current element is figure 4-2b can be associated with an image of opposite phase. The resistive component of mutual coupling between the current element and its image can be approximated by the expression

$$\begin{aligned} R_{12} &= R_{11} \quad \cos \pi b/\lambda \quad , \quad b \leq \lambda/2 \qquad (4\text{-}17) \\ &= R_{11} \quad \cos 2 \pi \; y/\lambda \end{aligned}$$

The resistive component of the self-impedance of the radiating element is assumed constant, having a value equal to its free space value

$$R_{11} = 80\pi^2 \,(dx/\lambda)^2 \qquad (4\text{-}18)$$

$$= 80\pi^2 \left(\frac{dy \ \tan\theta}{\lambda}\right)^2$$

The radiation resistance of the element-image combination is

$$R_r = R_{11} - R_{12} \qquad (4\text{-}19)$$

$$= 80\,\pi^2 \left(\frac{\sqrt{2} \ \tan\theta \sin\beta \ \ y/2 \ \ dy}{\lambda}\right)^2$$

The apparent length of the element-image pair is

$$dL = \sqrt{2} \ \ \tan\theta \sin\beta \ \ y/2 \ \ dy \qquad (4\text{-}20)$$

The effective length of the antenna is

$$L_e = \frac{1}{I_o} \int_0^{L(H)} I_{(s)} \ \ dL \qquad (4\text{-}21)$$

Equations (4-20) and (4-21) will now be used to estimate the component of radiation resistance associated with the horizontally polarized field for the tilted monopole. For the tilted monopole $\theta =$ constant and $y' = s \cos\theta$. (4-22)

Equations (4-20), (4-21), and (4-22) combine to give the following expression for the effective length of the tilted monopole.

$$L_e = \frac{\sqrt{2} \ \sin\theta}{\sin\beta H} \int_0^H \sin\beta(H-s) \quad \sin\left[\left(\frac{\beta\cos\theta}{2}\right) s\right] \ ds \qquad (4\text{-}23)$$

After integration, the expression for the effective length of the antenna becomes

$$\frac{L_e}{H_{eo}} = \frac{1}{\sqrt{2}} \frac{\beta H/2}{\sin^2\beta H/2} \left[ \begin{array}{l} \sin\beta H\left(\frac{\sin^2 B/2}{B/2} - \frac{\sin^2 A/2}{A/2}\right) \\ \\ + \cos\beta H\left(\frac{\sin B}{B} - \frac{\sin A}{A}\right) \end{array} \right] \sin\Theta$$

where $A = \beta H\left(\frac{2-\cos\Theta}{2}\right)$ (4-24)

$B = \beta H\left(\frac{2+\cos\Theta}{2}\right)$

The radiation resistance associated with this horizontally polarized radiation has the value

$$\left(\frac{R_r}{R_{ro}}\right)_H = \left(\frac{L_e}{H_{eo}}\right)^2 \qquad (4\text{-}25)$$

This radiation resistance is also plotted in figure 4-3 for a quarter-wave monopole. For a short monopole, the radiation resistance for the horizontally polarized field approaches zero rapidly.

$$\left(\frac{R_r}{R_{ro}}\right)_H \approx \frac{(\beta H)^2 (\sin 2\Theta)^2}{2} \quad ; \ \beta H \ll 1 \qquad (4\text{-}26)$$

It can be shown that the total power radiated will be the sum of the power radiated by the vertical current elements and by the horizontal current elements. The total radiation resistance is the sum of the radiation resistance for the two components.

$$R_r = (R_r)_V + (R_r)_H \qquad (4\text{-}27)$$

The total radiation resistance is plotted in figure 4-4

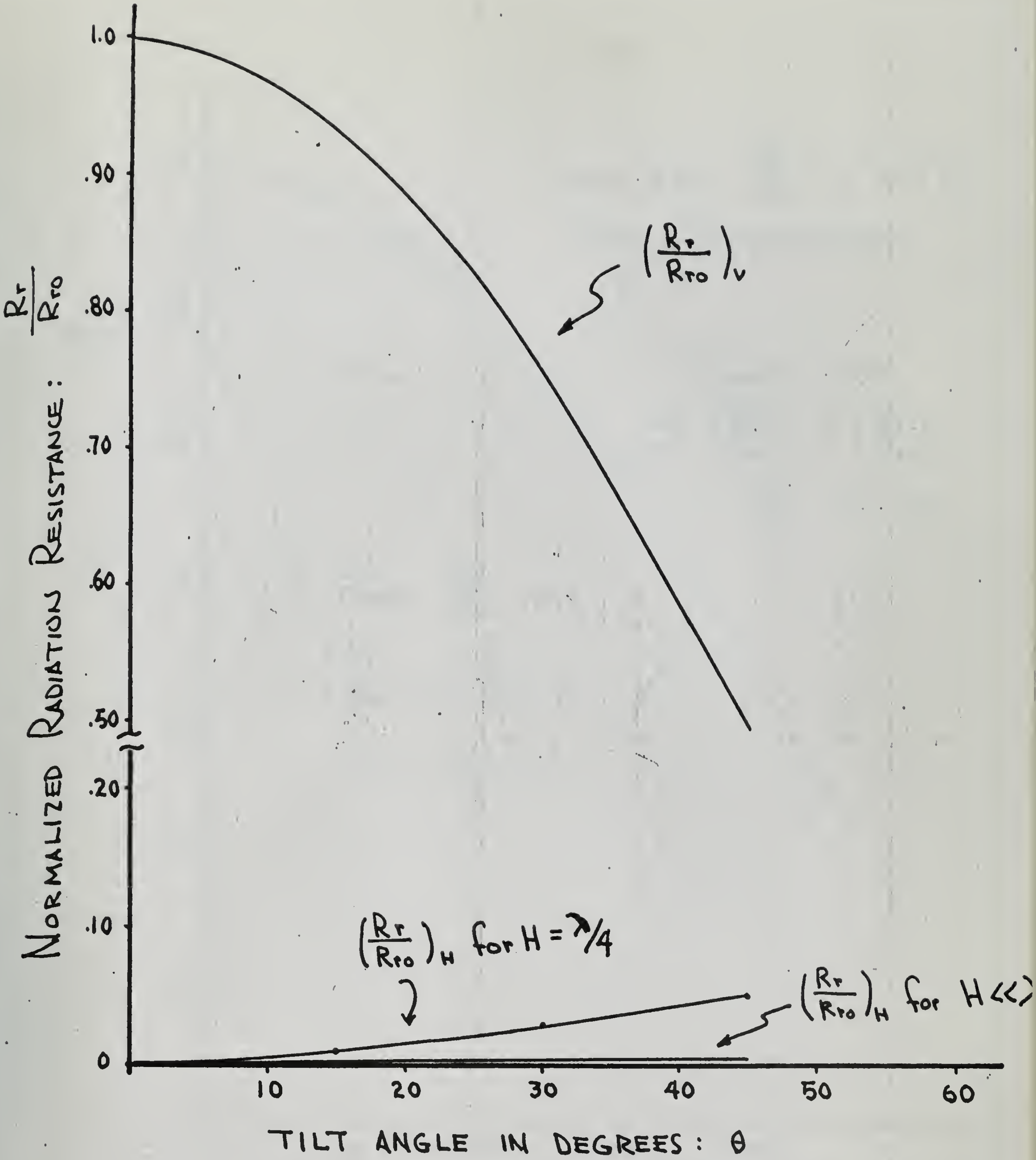



FIGURE 4-3. RESOLVED COMPONENTS OF RADIATION RESISTANCE FOR A TILTED MONOPOLE

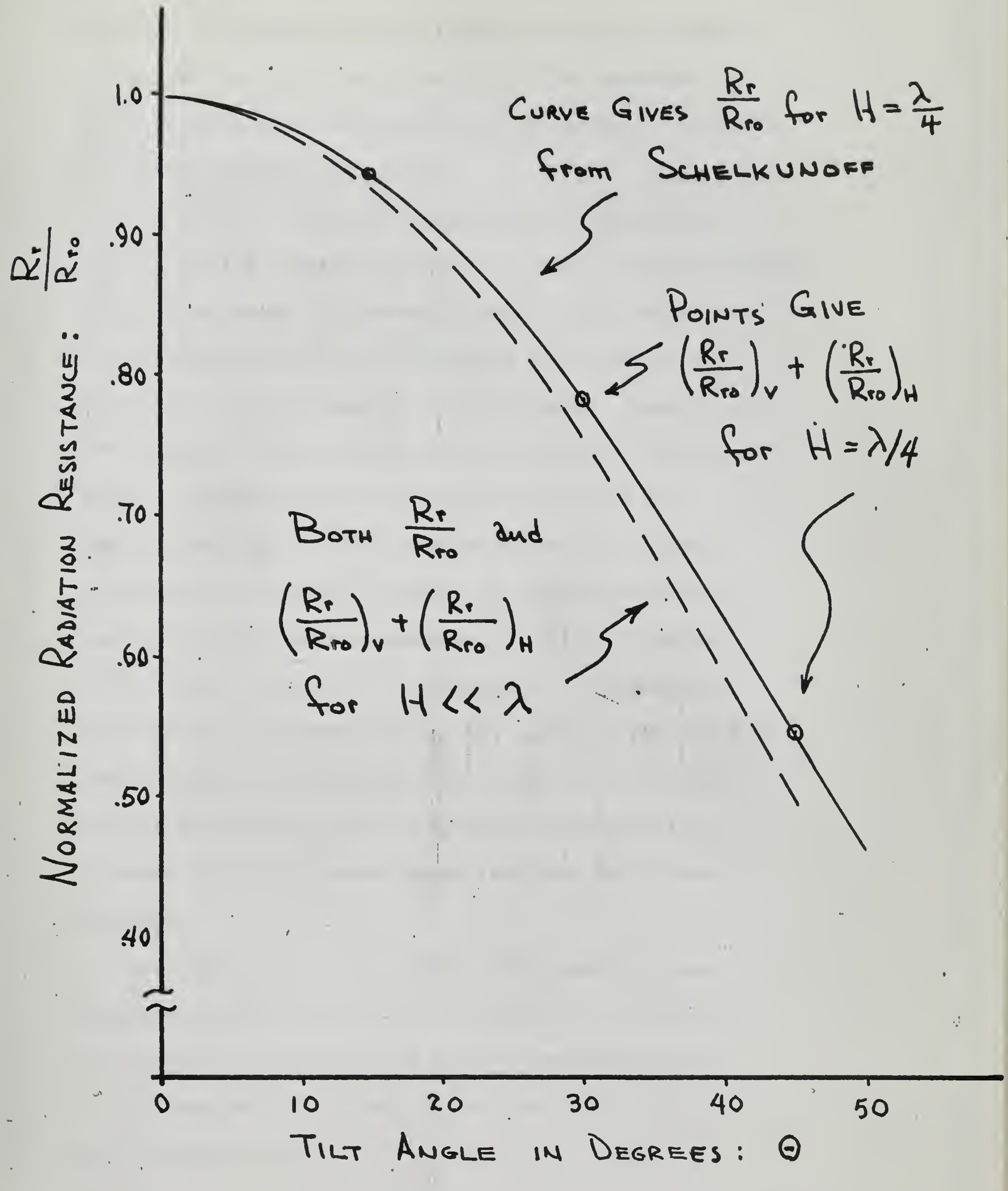



FIGURE 4-4. TOTAL RADIATION RESISTANCE FOR A TILTED MONOPOLE

where it is compared with the results obtained previously using a more exact method of analysis. The agreement is good, justifying the assumptions and the method of analysis.

5. Curved Antenna Calculations

The method of analysis described in the preceding section for the tilted monopole can be applied to any vertical curved wire antenna of reasonable shape; e.g., height increasing monotonically and dimensions of a fraction of a wavelength. The integrations for the general shaped wire are tedious and best performed by the use of a high-speed digital computer because the integral equations that describe the shape of the cable curves do not integrate directly and so numerical methods of integration must be used. For this reason an example of a simple curve was first chosen to simplify the mathematics. The general case will be described later in this section. The selected example illustrates the method of analysis for a curved antenna and provides a better quantitative estimate of blow-down on radiation resistance than does the tilted monopole.

The curve chosen to represent the high-wire antenna during blow-down is sketched in figure 5-1. It consists of a circular arc of length H which is vertical at the top-end and which is tilted off vertical by an angle at the bottom end.

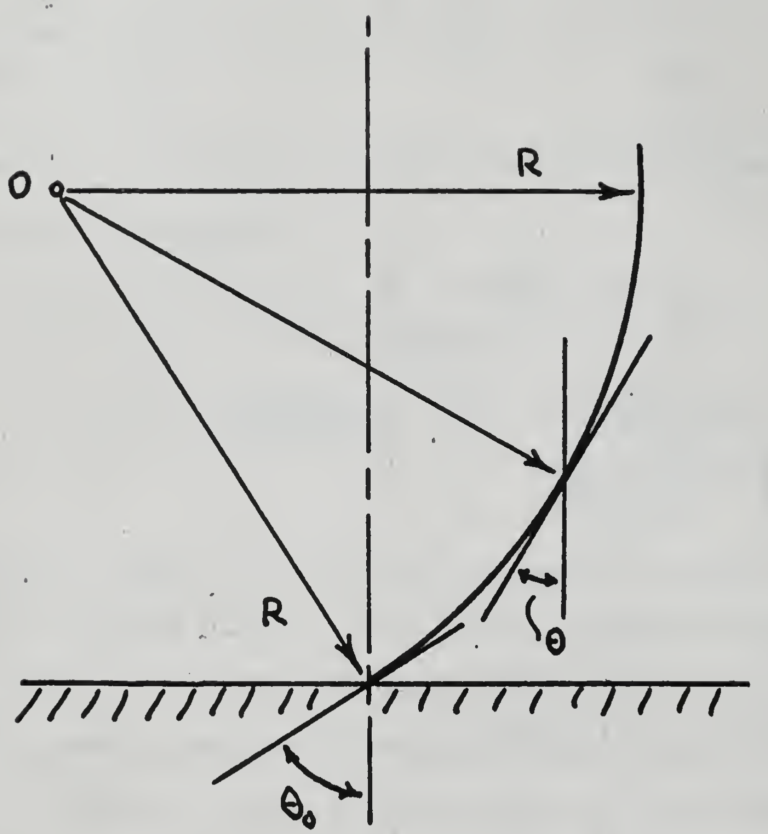



FIGURE 5-1. ASSUMED GEOMETRY FOR EXAMPLE CURVED WIRE ANTENNA

a. Vertical Polarization

To calculate the radiation resistance for the vertical component, use the expression for effective height given in equation (4-11). The sinusoidal xurrent distribution expressed in equation (4-12) is used. The local angle of tilt for the circular arc has the value

$$\theta = \frac{\theta_o}{H} (H - s) \tag{5-1}$$

Consequentily, the effective height of the curved antenna is given by the expression

$$H_e = \int_o^H \frac{\sin \beta (H - s) \cos \frac{\theta_o}{H} (H - s)}{\sin \beta H} ds$$

$$= H_{eo} \frac{(\beta H/2)}{\sin^2 (\beta H/2)} \left[ \frac{\sin^2 \left(\frac{\beta H - \theta_o}{2}\right)}{\left(\frac{\beta H - \theta_o}{2}\right)} + \frac{\sin^2 \left(\frac{\beta H + \theta_o}{2}\right)}{\left(\frac{\beta H + \theta_o}{2}\right)} \right]$$

The effective height of the antenna is plotted in figure 5-2 for several antenna lengths. Note that the effective height of the curved antenna decreases more rapidly than its own projected physical height but less repidly than the projected physical height of a tilted monopole having the same length and the same angle of tile at the base. The radiation resistance is proportional to the square of the effective height.

b. Horizontal Polarization

To calculate the radiation resistance of the horizontally

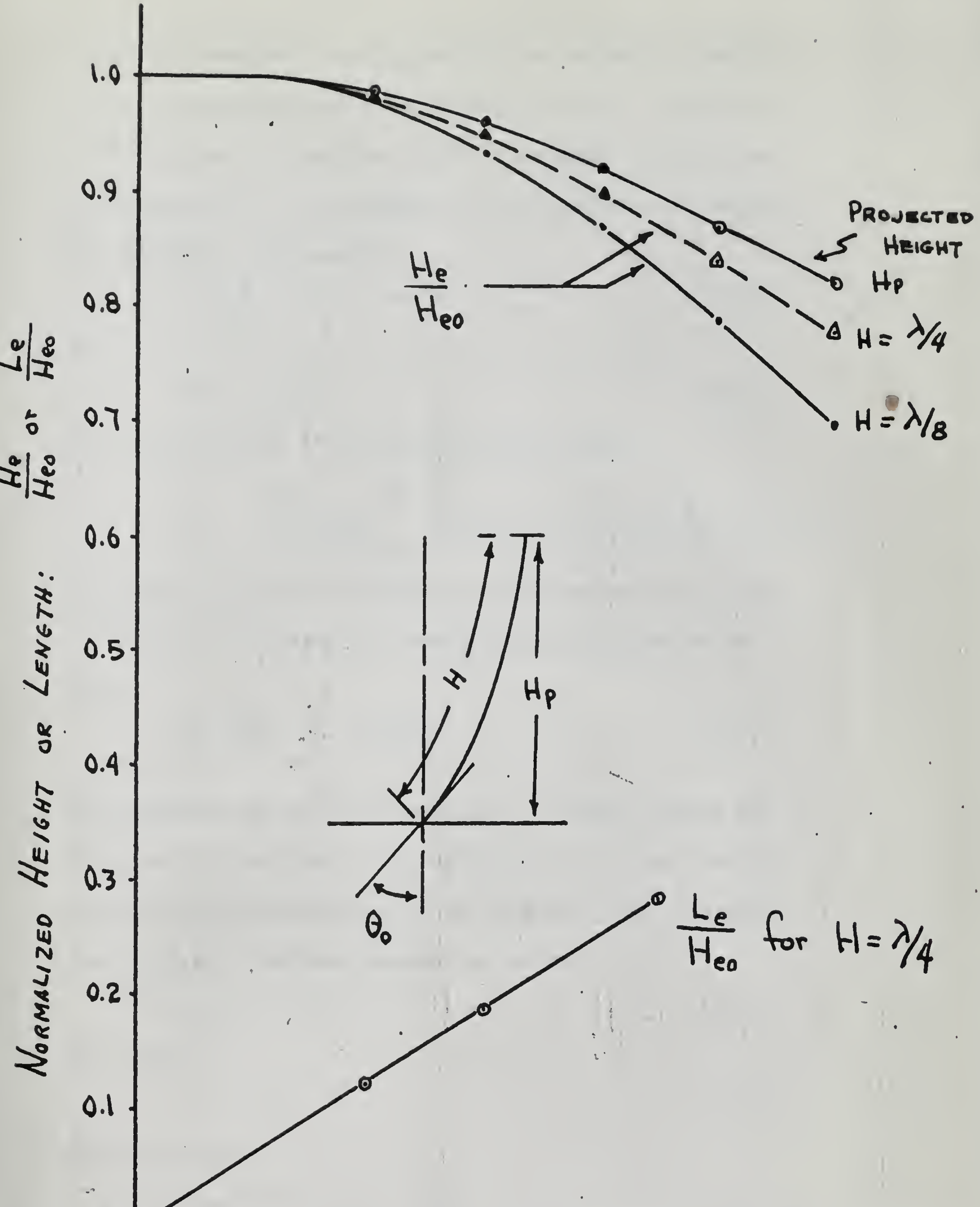



FIGURE 5-2. EFFECTIVE HEIGHT and LENGTH FOR THE EXAMPLE SIMPLE CURVED ANTENNA

polarized component, the expressions for effective length given in equations (4-20) and (4-21) are used. The sinusoidal current in equation (4-12) is assumed. In addition to equation (5-1), the geometry of the curcular arc imposes the following relationships

$$y = \frac{H}{\theta_0}(\sin\theta_0 - \sin\theta) \tag{5-3}$$

and

$$dy = \cos\theta \, ds \tag{5-4}$$

The expression for the effective length becomes

$$L_e = \frac{\sqrt{2}}{\sin \beta H}\int_0^H \sin\beta(H - s) \sin\theta \sin\frac{\beta y}{2} \, ds \tag{5-5}$$

Actually, equation (5-3) should be substituted into equation (5-5) for y. Instead, the simplifying assumption can be made

$$\sin\frac{\beta y}{2} \approx \sin\frac{\beta s}{2} \tag{5-6}$$

The assumtion is valid for small $\theta_0$. It shall be used for $\theta_0$ up to 0.75 radians. It is of interest to know in which direction the error introduced by equation (5-6) is tending for large $\theta_0$. Consider antenna for which

$$s \leq \lambda/4 \tag{5-7}$$

Then since

$$\beta y \leq \beta s \tag{5-8}$$

it follows that

$$\sin\frac{\beta y}{2} \leq \sin\frac{\beta s}{2} \tag{5-9}$$

The calculated values of effective length will be larger when the approximation is used than when it is not.

Proceeding with this approximation, the expression for effective length becomes

$$L_e \approx \frac{\sqrt{2}}{\sin\beta H}\int_0^H \sin\beta(H-s)\ \frac{\sin\theta_0(H-s)}{H}\ \frac{\sin\beta s}{2}\ ds \qquad (5\text{-}10)$$

$$= H_{e0}\frac{1}{\sqrt{2}}\left(\frac{\beta H/2}{\sin\beta H/2}\right)^2\left[\frac{\cos\frac{\beta H}{2} - \cos(\beta H-\theta_0)}{(\frac{3}{2}\beta H-\theta_0)(\beta\frac{H}{2}-\theta_0)} - \frac{\cos\frac{\beta H}{2} - \cos(\beta H+\theta_0)}{(-\frac{3}{2}\beta H+\theta_0)(\beta\frac{H}{2}+\theta_0)}\right] \qquad (5\text{-}11)$$

For short antennas, the effective length approaches zero rapidly.

$$Le \propto (\beta H)^2 \quad \text{and} \quad \beta H \ll 1 \qquad (5\text{-}12)$$

The worst case is for the largest antanna. Therefore, the effective length of the antenna is plotted in figure 5-3 only for that case of the quarter-wavelength antenna. Figure 5-3 gives the radiation resistance of the curved antenna as a function of tilt angle for the vertical component, the horizontal component, and the total field.

c. General Curved Antenna

In order to calculate the effective height of the curved antenna of general shape it can be seen from equation (4-11) that the local angle of tilt off the vertical ($\theta$) must be known as a function of the distance ( s ) along the curve. Equations (3-16) and (13-17) describe the general shape of

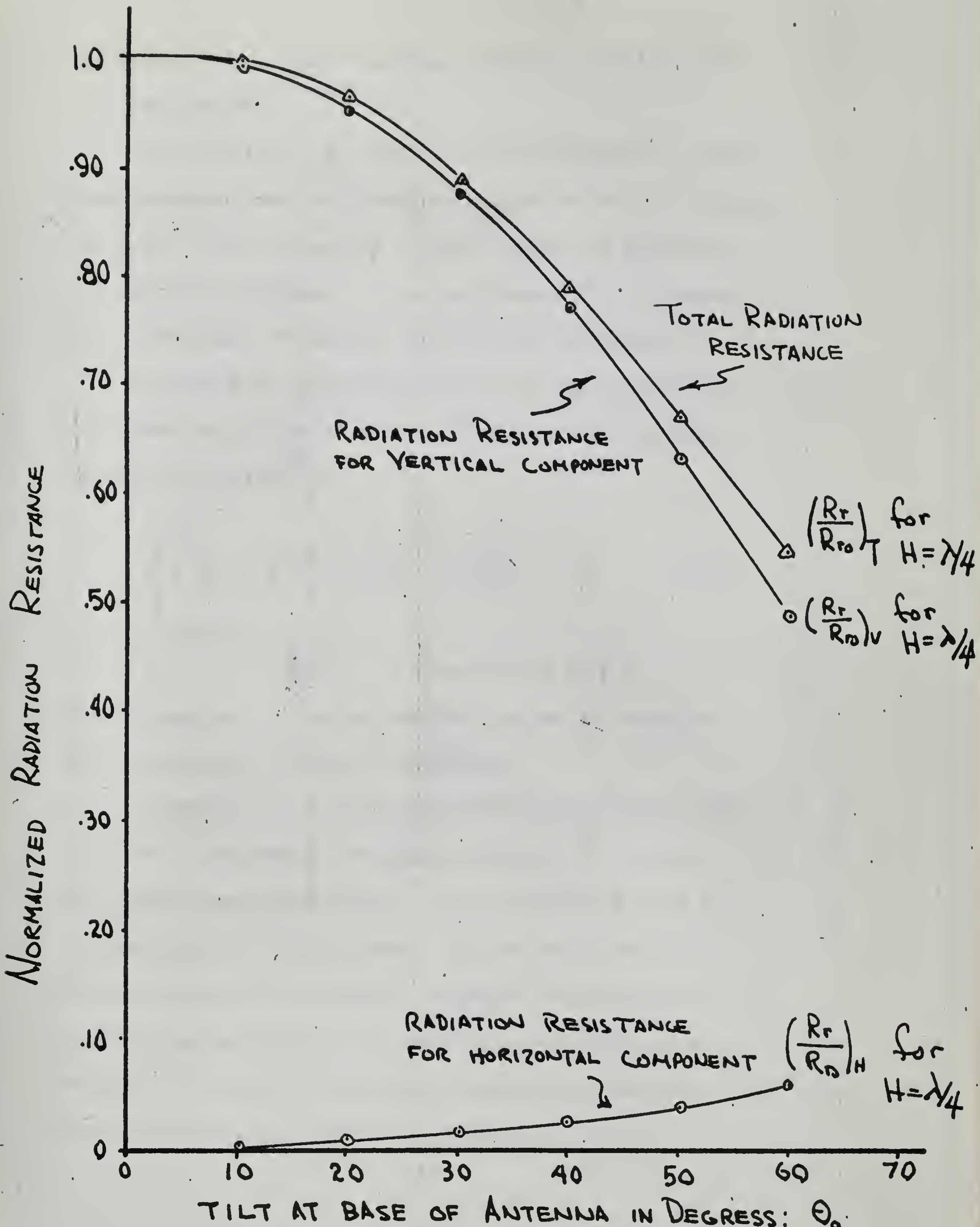



FIGURE 5-3 RADIATION RESISTANCE FOR THE EXAMPLE SIMPLE CURVED ANTENNA

the curved wire. These equations are used to evaluate $\theta$ as a function of s.

The functions $\sigma$, $\xi$, and $\eta$ are not integrable in closed form therefore numerical integration must be used. Although the function $\tau$ is integrable in closed form, see Appendix I, the expressions obtained are so complicated that in general it is preferable to compute $\tau$ by numerical integration.

The numerical integrations were made using Simpson's One-Third rule, together with a correction term involving fourth differences [6].

$$\int_{x_n-2h}^{x_n} y\,ds = \frac{h}{3}\left(y_n + 4y_{n-1} + y_{n-2}\right) - \frac{h}{90}\left(y_{n-1} + 6y_{n-2} - 4y_{n-3} + y_{n-4}\right) \qquad (5\text{-}13)$$

The integration interval of one-half degree was used for the calculations, i.e., h = 0.00872664.

The symbol, f, in equations (3-16a) and (3-17a) is the ratio of the tangential aerodynamic drag to the normal aerodynamic drag of the cable. This ratio has a value of approximately 0.01 for a smooth cylinder and a value of about 0.02 for cables having a roughness characteristic such as that exhibited by stranded cables. Because the values of the cable functions are relatively insensitive to changes in this parameter and because the cable is

assumed to be smooth, the value of $f = 0.01$ will be used in all the calculations of cable shape.

The cable under consideration is assumed to be of compound construction with an inner strength member of synthetic fiber, a layer of aluminum wire as the electrical conductor, and an external synthetic jacket. This type of cable construction is feasible with a strength-to-weight ratio much greater than that of a steel cable. The other assumed physical characteristics of the cable are:

| | |
|---|---|
| Diameter | 0.72 inches |
| Unit weight | 0.19 pounds per foot |
| Breaking strength | 10,000 pounds |

Within the assumptions described above, the following parameters are involved in the specification of the cable configuration:

a. The cable angle at the top
b. The cable angle at the bottom
c. The altitude at the top
d. The horizontal displacement of the top
e. The tension at the top
f. The tension at the bottom
g. The cable length
h. The cable diameter
i. The cable weight
j. The wind velocity

Reference 1. proves that for a given cable and wind (parameters h., i., and j.) the shape of the cable is completely specified by specifying any three of the remaining parameters (a. through g.). The present problem

of determining antenna shape is therefore solved by selecting the above cable parameters and various winds and specifying the cable length, the cable angle at the top, and the tension at the top.

6. Results and Conclusions

The effect of blow-down is to decrease the radiation resistance of the antenna and to introduce a horizontally polarized component of radiation. The radiation resistance decreases faster than would be attributed to a simple reduction in physical height. Because the antenna is curved under a condition of blow-down, the larger tilt angles at the lower end of the antenna are emphasized by the larger radiating currents at the lower end.

The horizontal component of radiation helps suppress the decrease in radiation resistance as blow-down occurs. But the horizontal component of the radiation is not useful. It is beamed vertically, it suffers large propagation losses, and it is cross polarized to vertical receiving antennas. It is more correctly considered a power loss than useful radiation.

It was found possible to characterize the vertical and horizontal polarizations by separate radiation resistances. For long range VLF surface communications, only the radiation resistance associated with the vertical polarization should

be considered useful radiation resistance. The resistance assocaited with the horizontal polarization should be considered a loss resistance. Therefore, blow-down decreases the radiation efficiency of the antenna by decreasing the useful radiation resistance while introducing an additional loss resistance.

The antenna curvature during blow-down is a function of the many parameters listed in section 5, and the effective height of this antenna will be a function of its curved physical shape.

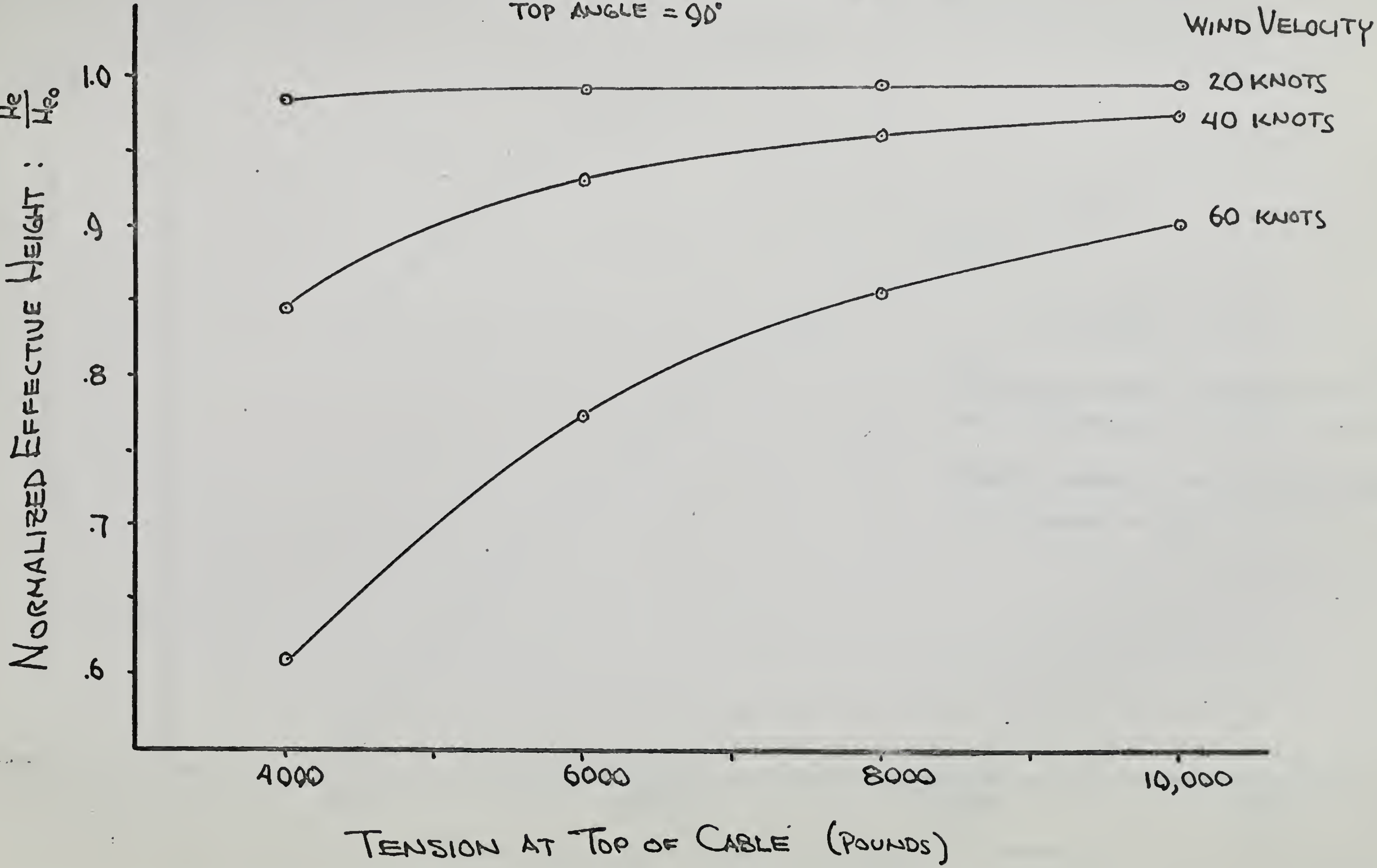
By varying the parameters one at a time while holding the others constant will give examples of how each factor effects the effective height of the antenna during blow-down. In figures 6-1a,b and 6-2a,b) the normalized effective height and the tilt angle at the base of the antenna are plotted as a function of the cable tension at the top of the antenna for several values of cable length and wind velocity. As the top tension is increased the cable becomes less curved and so the tilt angle at the lower end of the antenna is decreased. Therefore the effective beight of the antenna will be increased for increased values of tension, but the top tension is limited by the lift available from the supporting vehicle and by the breaking strength of the cable.

Figures 6-3 and 6-4 show the effective height and bottom

FIGURE 6-1a. NORMALIZED EFFECTIVE HEIGHT AS A FUNTION OF TOP TENSION

CABLE LENGTH = 7000 FEET

TOP ANGLE = 90°



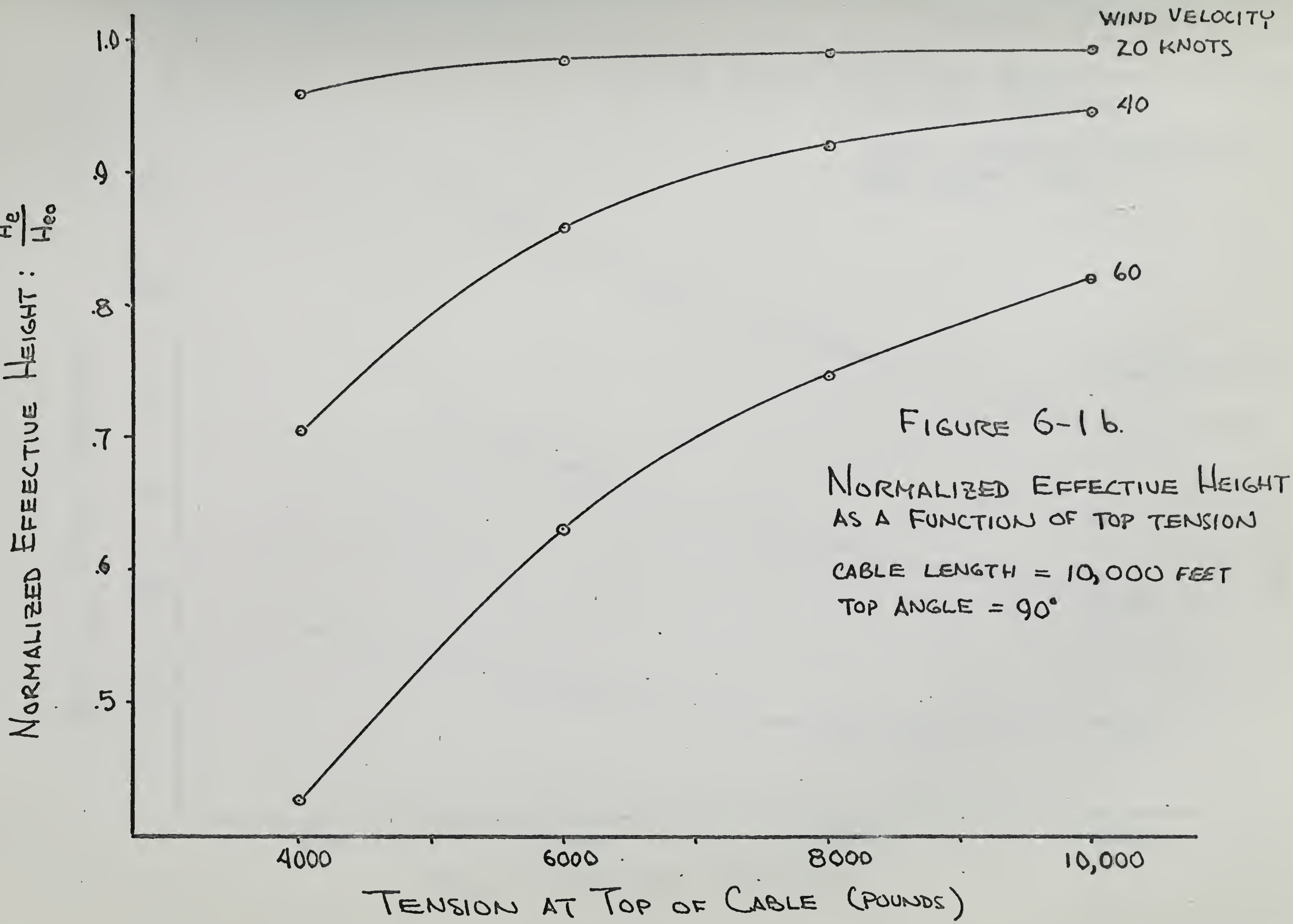



FIGURE 6-1 b.

NORMALIZED EFFECTIVE HEIGHT AS A FUNCTION OF TOP TENSION

CABLE LENGTH = 10,000 FEET

TOP ANGLE = 90°

Figure 6-2a. Bottom Angle as a function of Top Tension

Cable Length = 7,000 feet
Top Angle = 90°

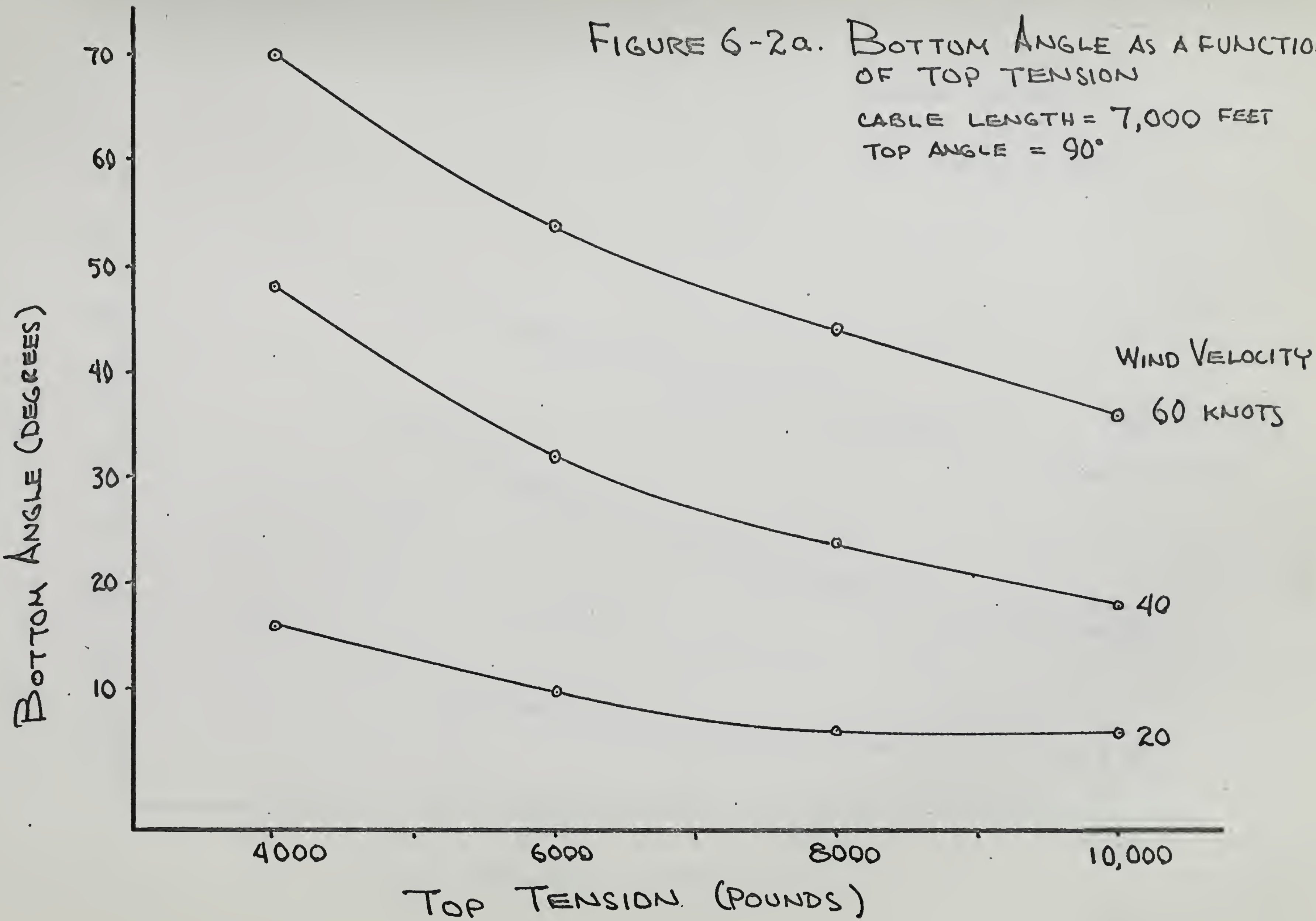
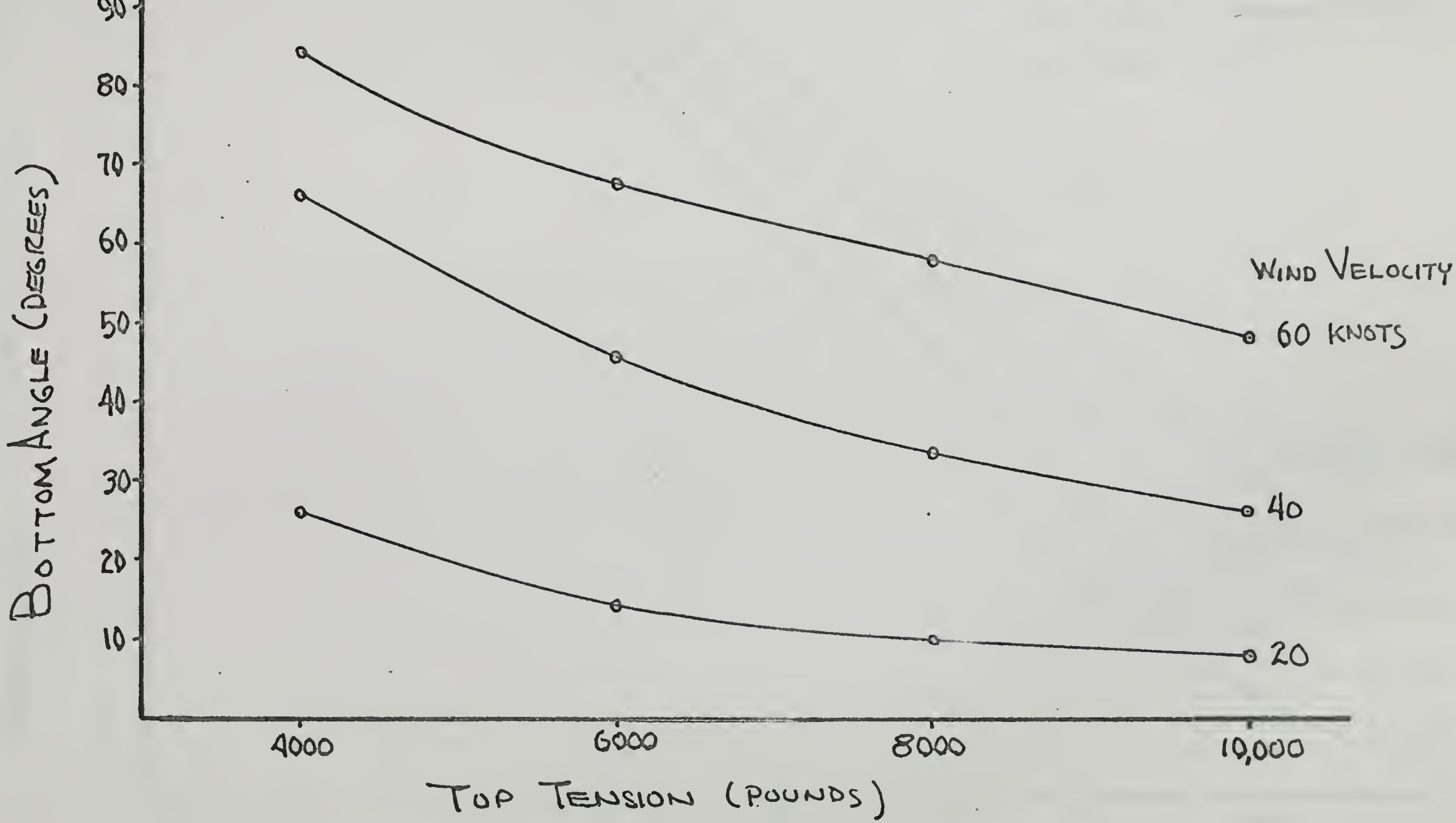


Figure 6-2b. Bottom Angle as a Function of Top Tension

Cable Length = 10,000 feet
Top Angle = 90°



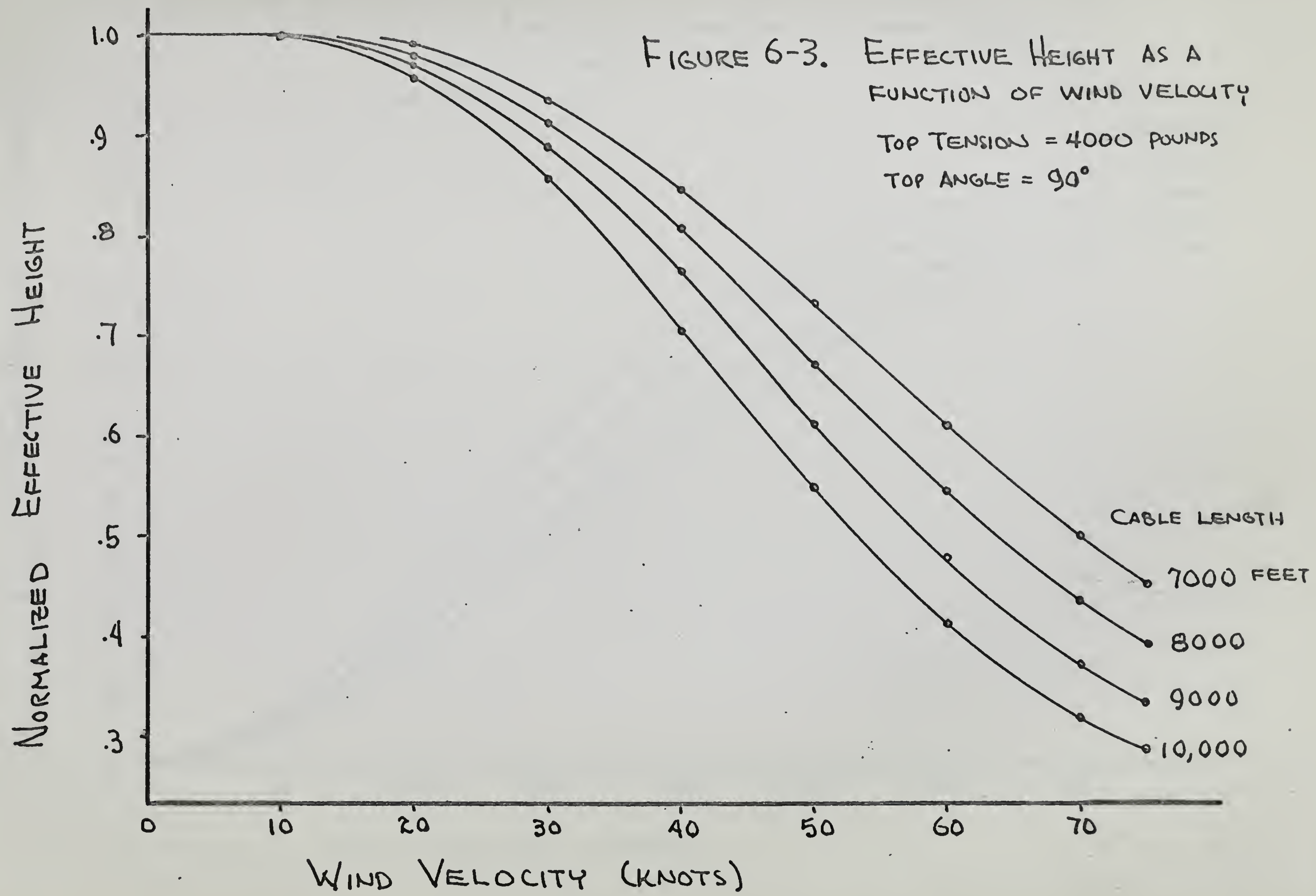
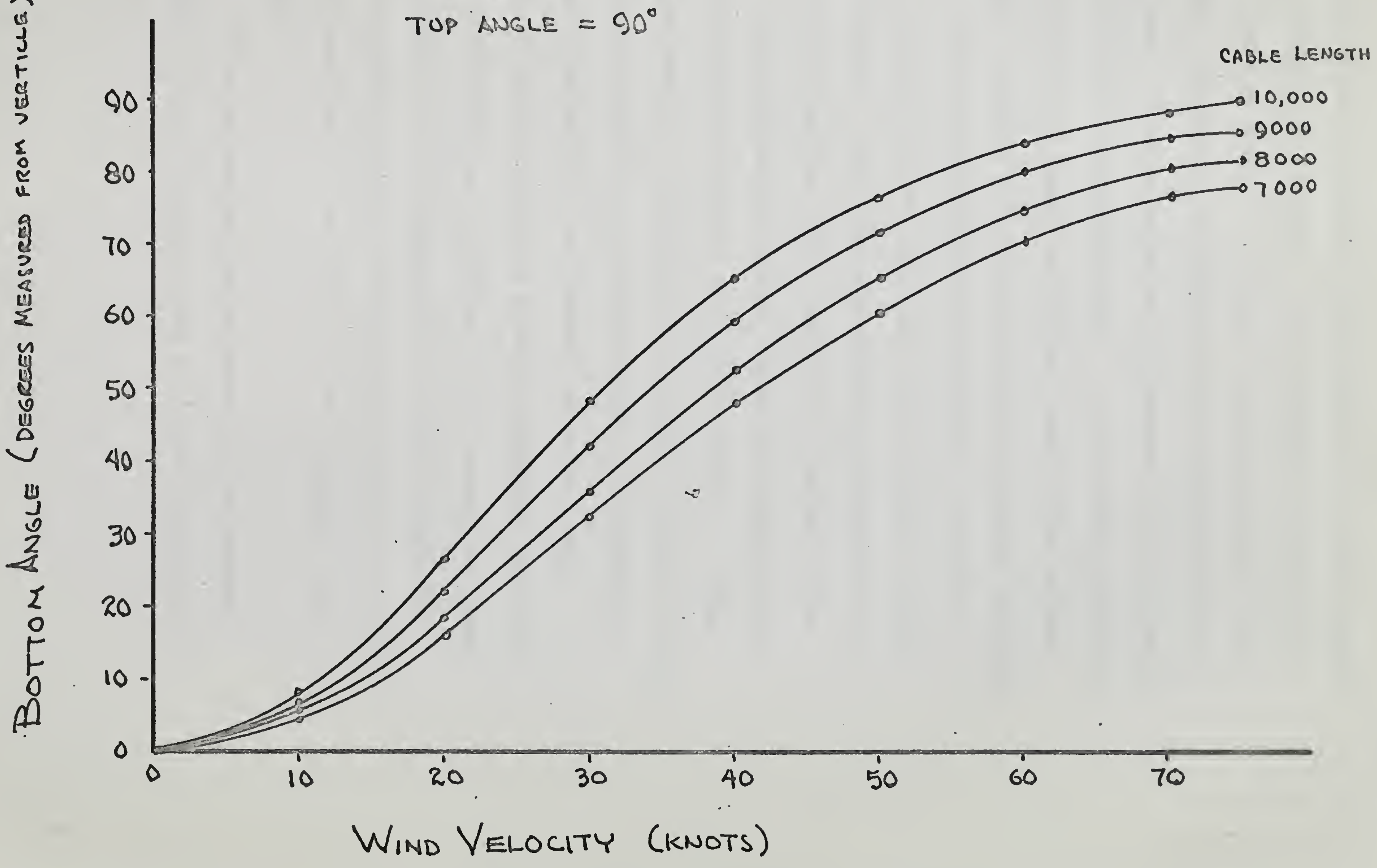



Figure 6-3. Effective Height as a Function of Wind Velocity

FIGURE 6-4. BOTTOM ANGLE AS A FUNCTION OF WIND VELOCITY
TOP TENSION = 4000 POUNDS
TOP ANGLE = 90°



tilt angle as a function of wind velocity for several values of physical cable length. In this example the top tension and the tilt angle at the top of the antenna are held constant. As expected, an increase in wind velocity results in a decreased effective height and an increase in the tilt angle at the base of the antenna cable. It can be seen from these curves that the longer the physical length of the antenna cable the greater will be the detrimental effect of the wind.

The effective height of the antenna as a function of the tilt angle at the top of the cable is shown in figures (6-5a thru d). This top tilt angle may be induced in the cable by flying the lifting vehicle into the wind. This top tilt angle may take values between the vertical angle and the critical angle as described in section 3. These curves show that a top angle of 90 degrees (vertical ) is the worst case and that the effective height is increased by causing the top angle to approach the critical angle. A table of critical angles is given in Appendix II and a plot of critical angle as a function of wind velocity is given in figure 6-8. The improvement of effective height due to a top angle near the critical angle is greater for higher wind velocities and for shorter values of physical cable length.

The actual physical curves assumed by the antenna cable for several typical values of physical length, top

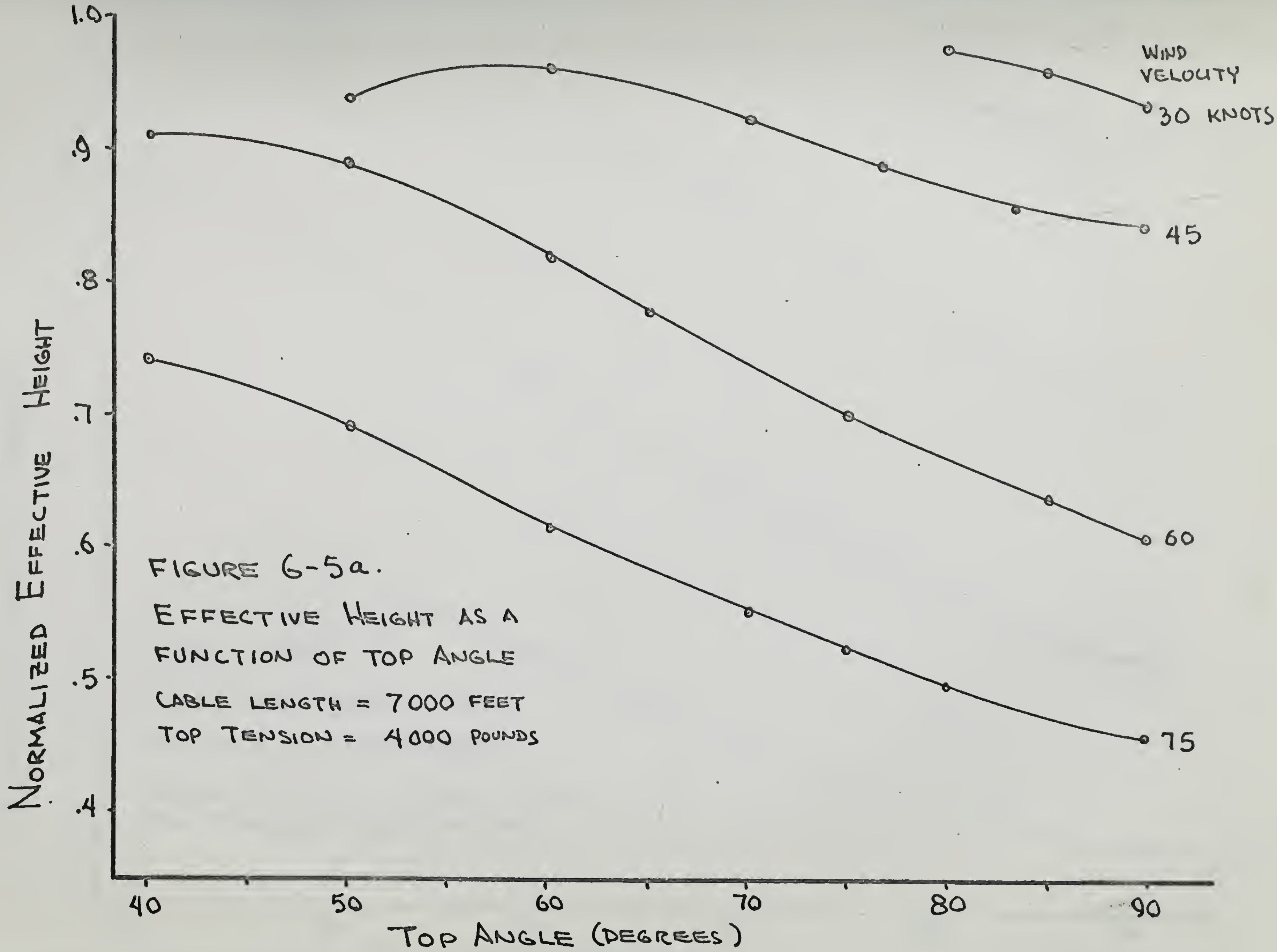



FIGURE 6-5a.
EFFECTIVE HEIGHT AS A FUNCTION OF TOP ANGLE
CABLE LENGTH = 7000 FEET
TOP TENSION = 4000 POUNDS

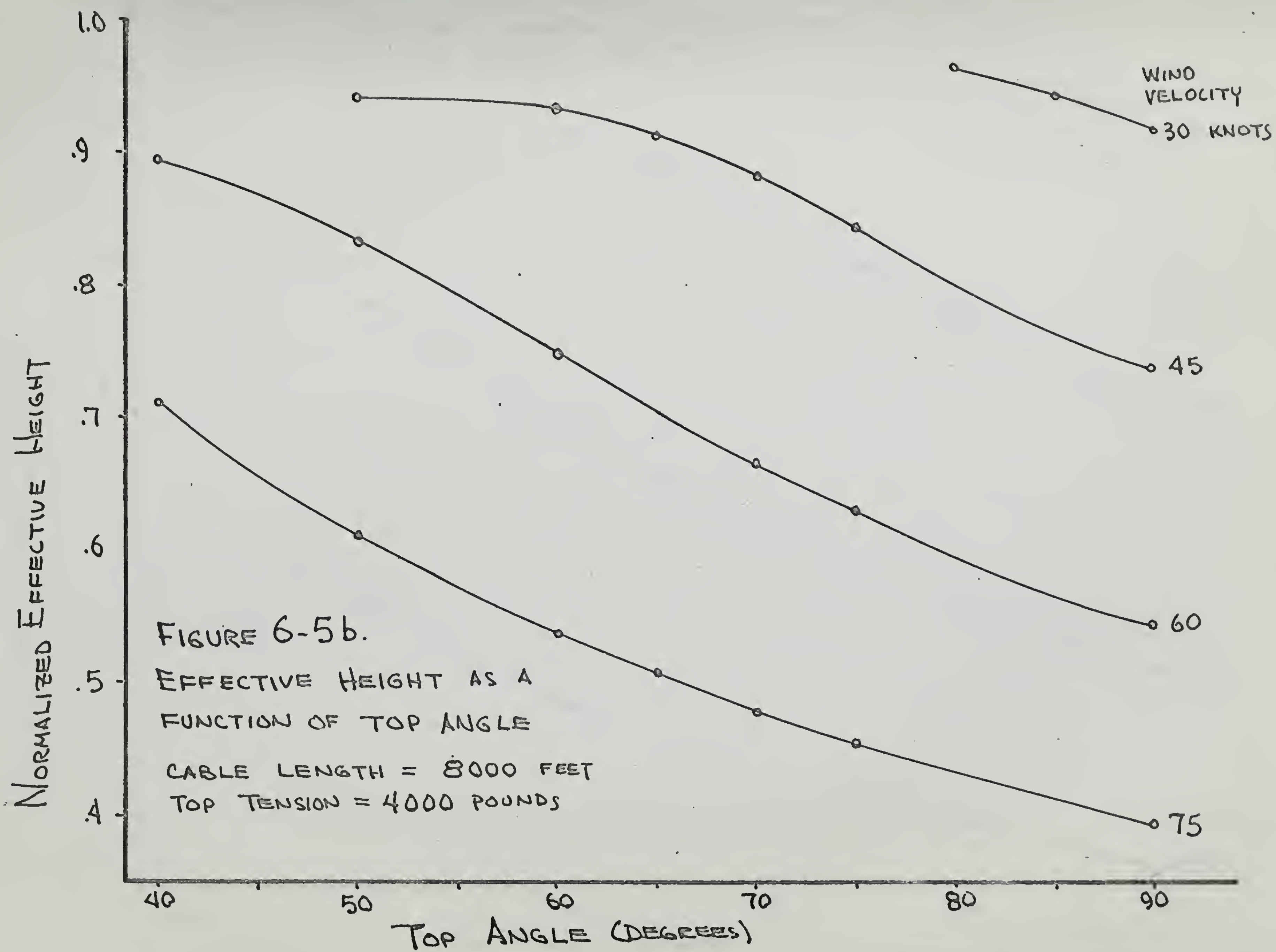



FIGURE 6-5b.
EFFECTIVE HEIGHT AS A FUNCTION OF TOP ANGLE

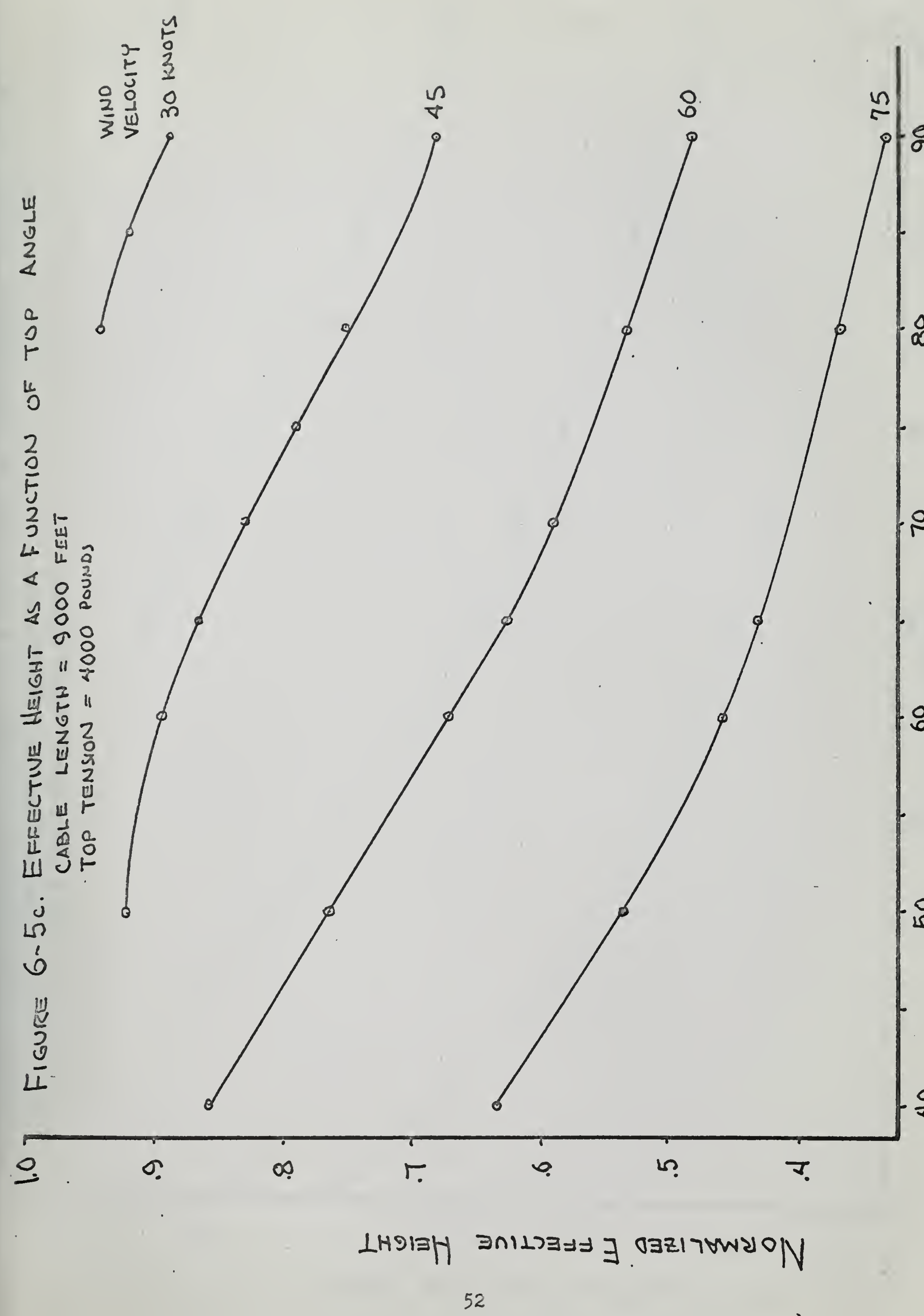
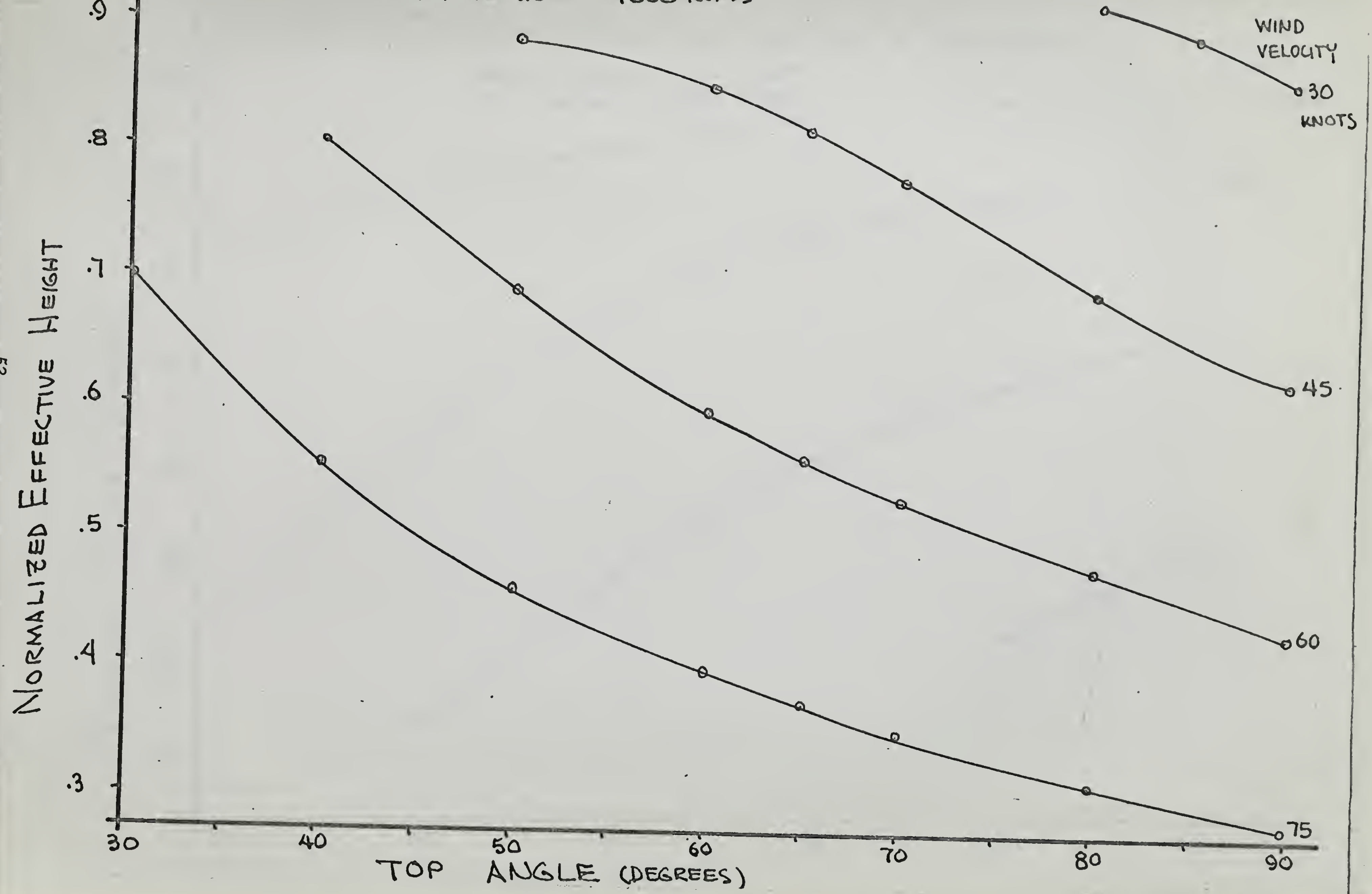



Figure 6-5c. Effective Height as a Function of Top Angle

Figure 6-5d. Effective Height as a Function of Top Angle
Cable Length = 10,000 Feet
Top Tension = 4000 Pounds



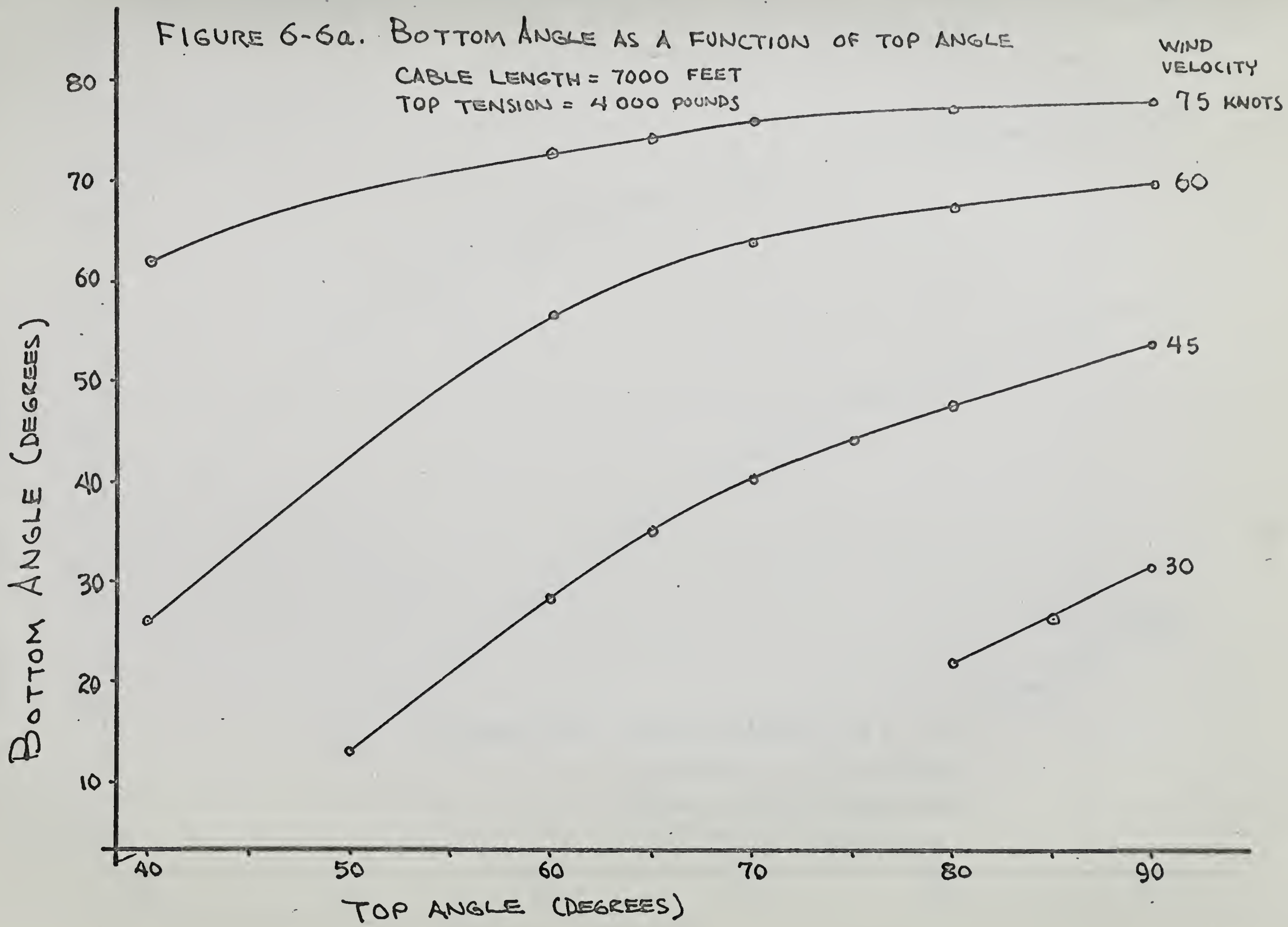



FIGURE 6-6a. BOTTOM ANGLE AS A FUNCTION OF TOP ANGLE

CABLE LENGTH = 7000 FEET

TOP TENSION = 4 000 POUNDS

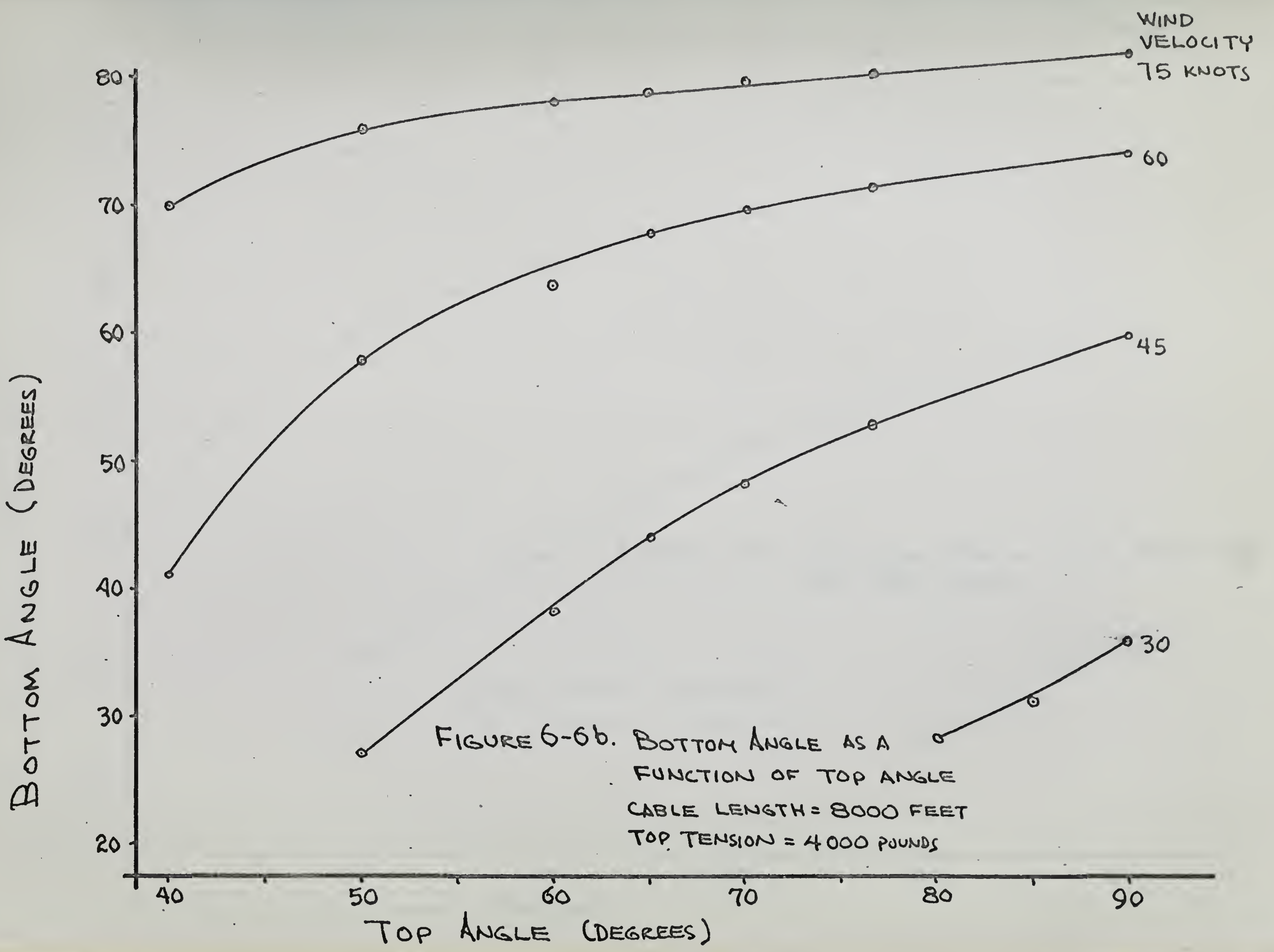



FIGURE 6-6b. BOTTOM ANGLE AS A FUNCTION OF TOP ANGLE

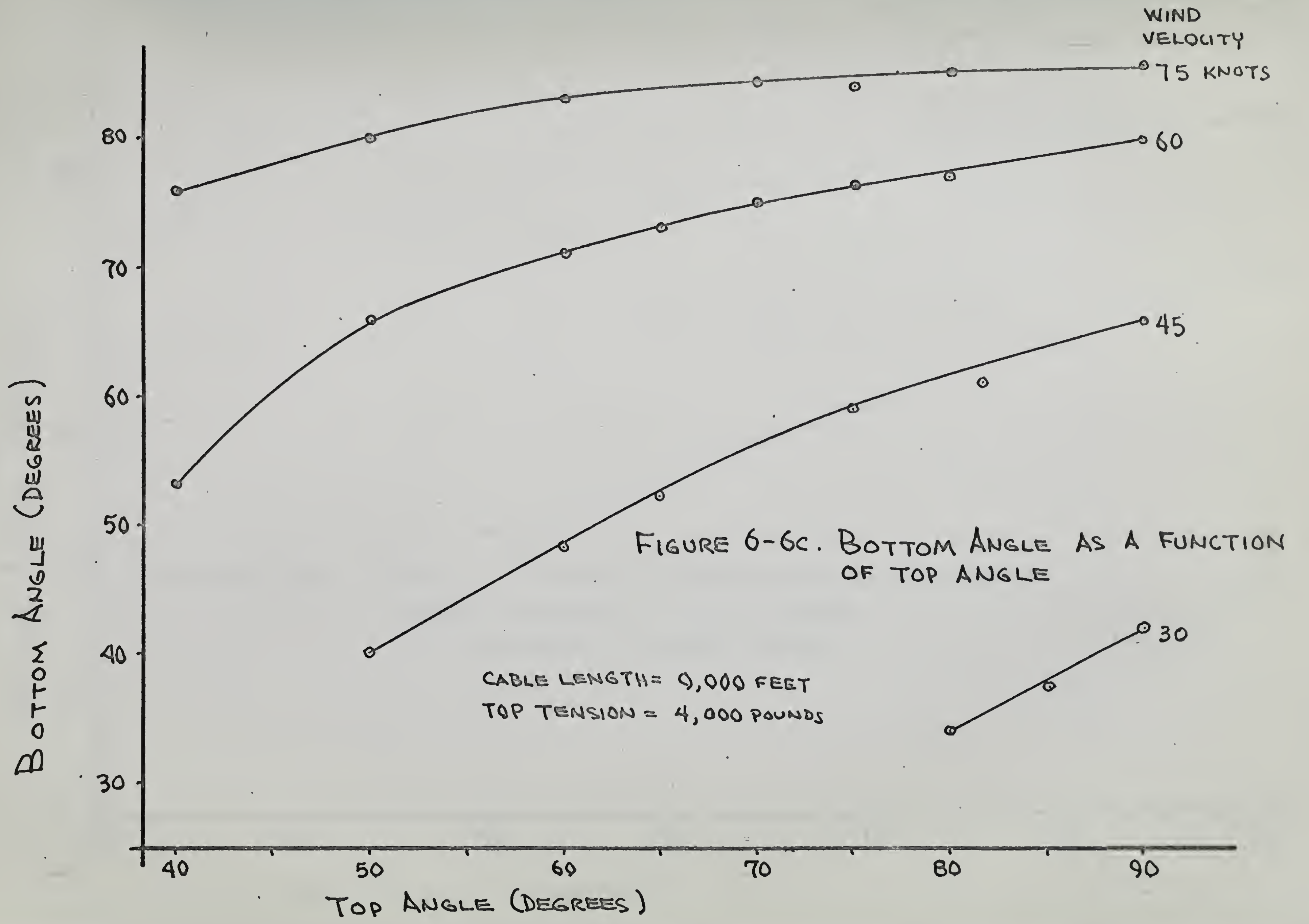



FIGURE 6-6c. BOTTOM ANGLE AS A FUNCTION OF TOP ANGLE

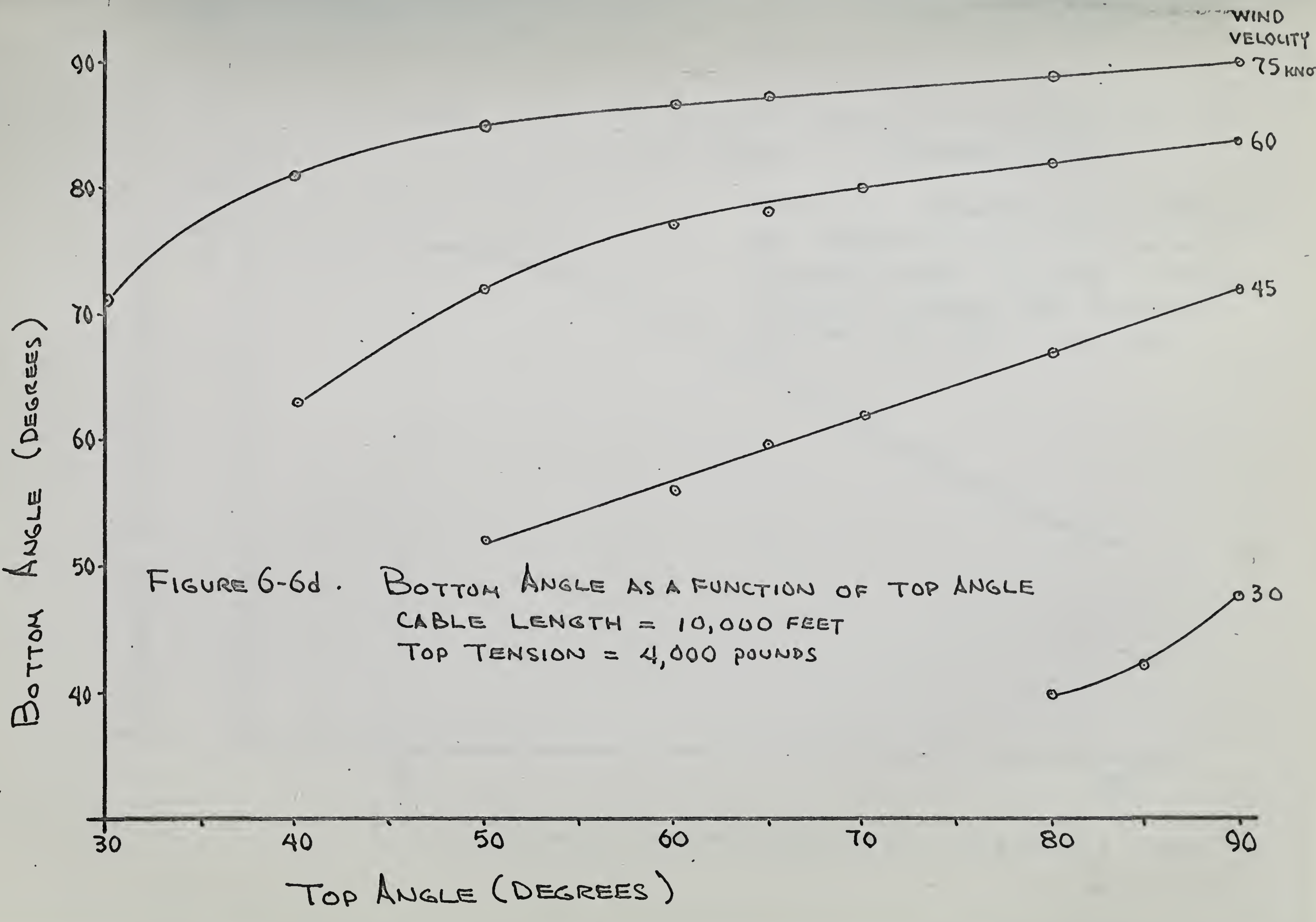
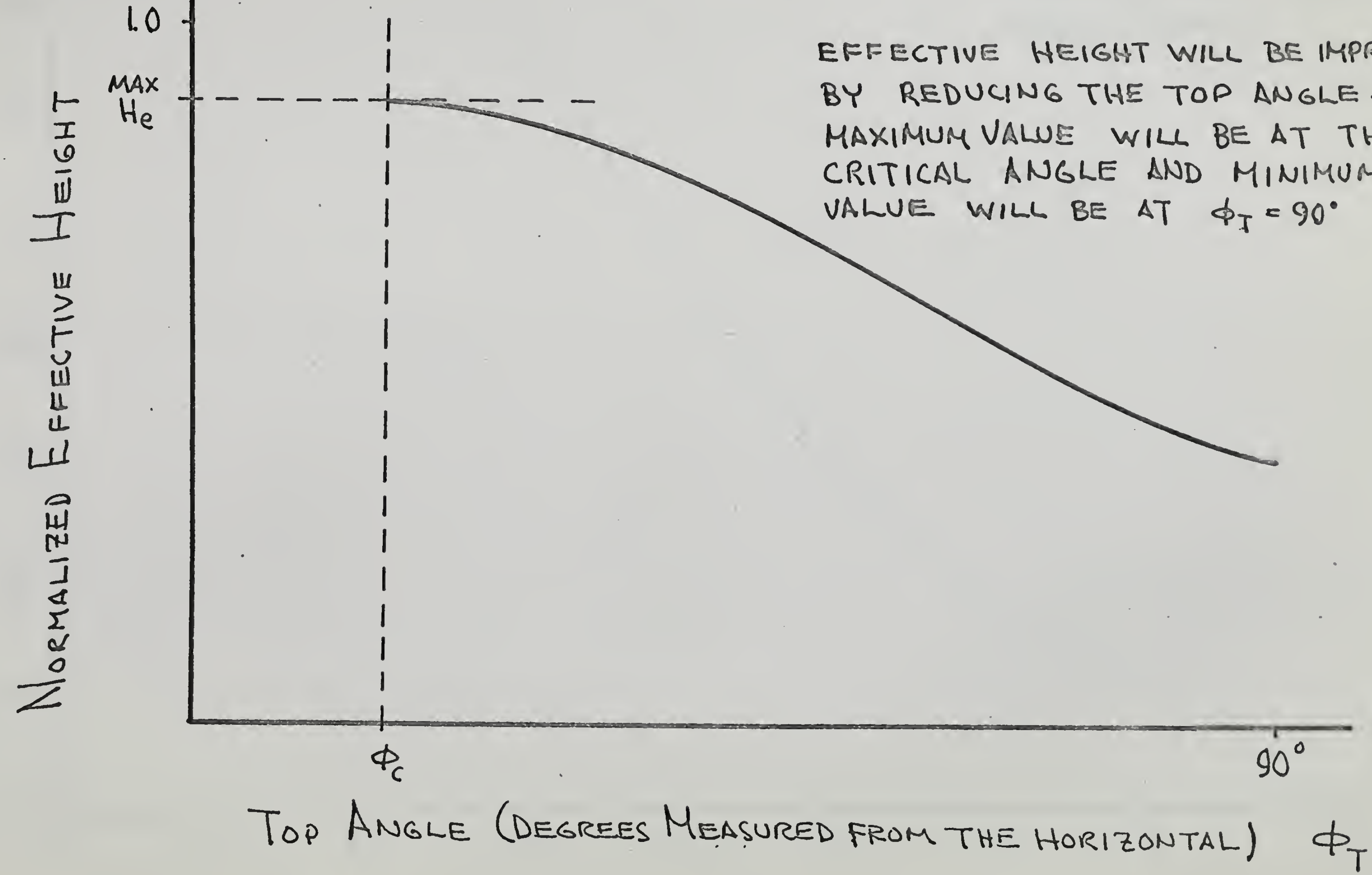



FIGURE 6-6d. BOTTOM ANGLE AS A FUNCTION OF TOP ANGLE
CABLE LENGTH = 10,000 FEET
TOP TENSION = 4,000 POUNDS

FIGURE 6-7. EFFECTIVE HEIGHT AS A FUNCTION OF TOP ANGLE - GENERAL CASE



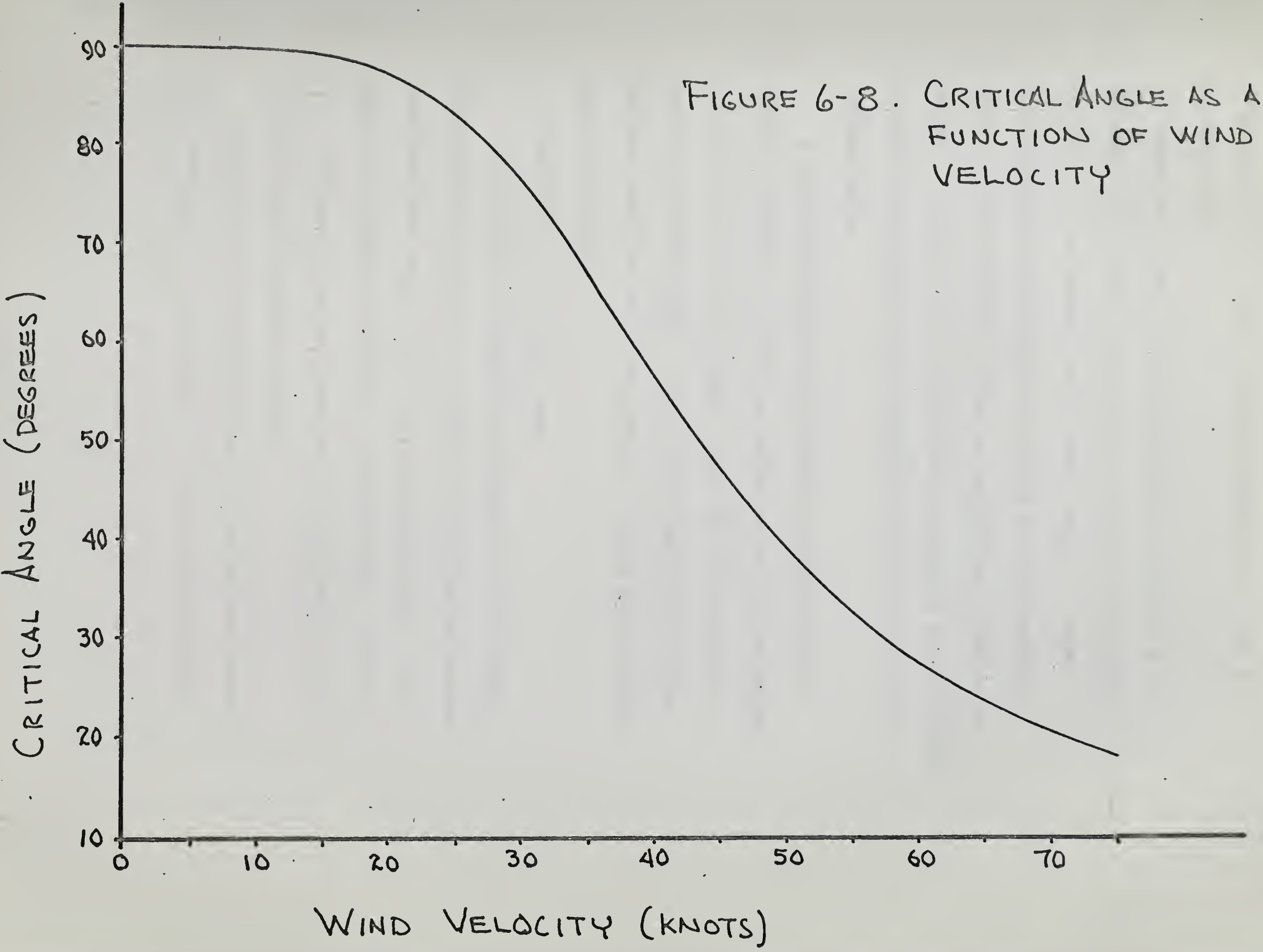



Figure 6-8. Critical Angle as a Function of Wind Velocity

tension, top tilt angle, and wind velocity are given in Appendix III.

To summarize, it may be concluded that an increase in top tension will increase the effective height of the antenna. In fact, if the support vehicle could apply an amount of lift great enough the cable could be pulled vertical under any wind condition, but this method of maximizing the effective height is limited by the lifting force available from practical support vehicles and by the breaking strength of a practical antenna cable.

A practical method of optimising the cable shape in order to maximize the effective height is to fly the support vehicle so that the tilt angle at the top of the cable is near the critical angle. This critical angle will be the smallest angle the cable can take for a given wind velocity and cable tension.

7. Suggestions for further study

In this report the wind was assumed to be of constant velocity and direction. It would be of interest to develope equations that would describe the cable shape for a wind that is a function of altitude. The wind could be analyzed as a continuously varying function or as stratums in which the wind could be assumed constant.

The current distribution along the antenna was assumed to be sinusoidal in this report. A useful study would be

an investigation into the effects of the antenna cable curvature on the actual current distribution. The effective height is a function of the current distribution, therefore before accurate absolute values of effective height may be determined the current distribution must be accurately known.

A third area of study would be to develope a general expression for the antenna's effective height. Perhaps the calculus of variations could be employed to develope this general model.

## APPENDIX I

## THE INTEGRATION OF ln $\tau$

From equation (3-16a) for Quadrant 1

$$\ln\tau = \int_{\pi/2}^{\phi} \frac{f + w \sin\phi}{-\sin^2\phi + w \cos\phi}\, d\phi$$

The denominator of the intregrand can be factored,

$$-\sin^2\phi + w\cos\phi = (\cos\phi - \cos\phi_c)(\cos\phi + \sec\phi_c)$$

since $w = \sec\phi_c - \cos\phi_c$ (from reference 1.)

so that $1/(-\sin^2\phi + w\cos\phi)$ can be written

$$\frac{1}{-\sin^2\phi + w\cos\phi} = \frac{1}{\sec\phi_c + \cos\phi_c}\left[\frac{1}{\cos\phi - \cos\phi_c} - \frac{1}{\cos\phi + \sec\phi_c}\right]$$

Hence

$$\ln\tau = \frac{1}{\sec\phi_c + \cos\phi_c}\left\{ f\left[\int_{\pi/2} \frac{d\phi}{\cos\phi - \cos\phi_c} - \int_{\pi/2} \frac{d\phi}{\cos\phi + \sec\phi_c}\right] + w\left[\int_{\pi/2}^{\phi} \frac{\sin\phi\, d\phi}{\cos\phi - \cos\phi_c} - \int_{\pi/2}^{\phi} \frac{\sin\phi\, d\phi}{\cos\phi + \sec\phi_c}\right]\right\}$$

The integrals that appear in this equation are listed in Pierce's Tables of Integrals[4].

$$\ln\tau = \frac{1}{\sec\phi_c + \cos\phi_c}\left\{ f\left[\csc\phi_c \ln\left(\frac{\tan\frac{\phi}{2} + \tan\frac{\phi_c}{2}}{\tan\frac{\phi}{2} - \tan\frac{\phi_c}{2}}\right) - 2\cot\phi_c \tan^{-1}\left(\tan\frac{\phi_c}{2}\tan\frac{\phi}{2}\right)\right] + w\ln\left(\frac{\cos\phi + \sec\phi_c}{\cos\phi - \cos\phi_c}\right)\right\}\Bigg|_{\pi/2}^{\phi}$$

From equation (3-17a) for Quadrant 2

$$\ln\tau = \int_{\pi/2}^{\phi} \frac{-f + w\sin\phi}{-\sin^2\phi + w\cos\phi}\, d\phi$$

This is of the same form as the expression for Quadrant 1 except for the sign of the constant f. The integral will therefore be of the form

$$\ln \tau = \frac{1}{\sec\phi_c + \cos\phi_c}\left\{ f\left[2\cot\phi_c \tan^{-1}\left(\tan\frac{\phi_c}{2}\tan\frac{\phi}{2}\right) - \csc\phi_c \ln\left(\frac{\tan\frac{\phi}{2} + \tan\frac{\phi_c}{2}}{\tan\frac{\phi}{2} - \tan\frac{\phi_c}{2}}\right)\right] + w\ln\left(\frac{\cos\phi + \sec\phi_c}{\cos\phi - \cos\phi_c}\right)\right\}\Bigg|_{\pi/2}^{\phi}$$

## APPENDIX II

### The Critical Angle

The values of the critical angle as described in section 3 are the roots of the equation $Q(\phi_c) = 0$. When the cable is simply trailed without being anchored at its bottom end, the configuration of the cable could be any straight line inclined to the stream at such an angle $\phi = \phi_c$. In order that there be a completely unique solution for this condition it is required that the equation $Q(\phi_c) = 0$ have no more than one root in the range $0 \leq \phi_c \leq \pi$. With this condition satisfied the equation

$$R \sin^2\phi_c - W \cos\phi_c = 0 \qquad (1)$$

is satisfied. Substituting $\sin^2\phi_c = 1 - \cos^2\phi_c$ and dividing by R

$$\cos^2\phi_c + \frac{W}{R} \cos\phi_c - 1 = 0 \qquad (2)$$

Hence

$$\cos\phi_c = -\frac{W}{2R} + \sqrt{\left(\frac{W}{2R}\right)^2 + 1} \qquad (3)$$

and

$$\cos\phi_c = -\frac{W}{2R} - \sqrt{\left(\frac{W}{2R}\right)^2 + 1} \qquad (4)$$

Equation (4) applies for the case that the cable has negative weight and may therefore be discarded. The weight per unit length of the cable, W, is constant, but the drag per unit length of cable when the cable is normal to the stream, R, is a function of wind velocity, and therefore the critical

angle is a function of wind velocity.

Using the cable characteristics listed in section 5c and the expression

$$R = \frac{1}{2} \rho v^2 d \; C_R$$

where

$\rho =$ mass density of air (0.002378 slugs per cubic foot)

$v =$ velocity of stream (feet per second)

$d =$ diameter of cable (feet)

$C_R =$ standard engineering constant (1.2)

Values of critical angle are tabulated for a wide range of wind velocities in Table One.

TABLE ONE

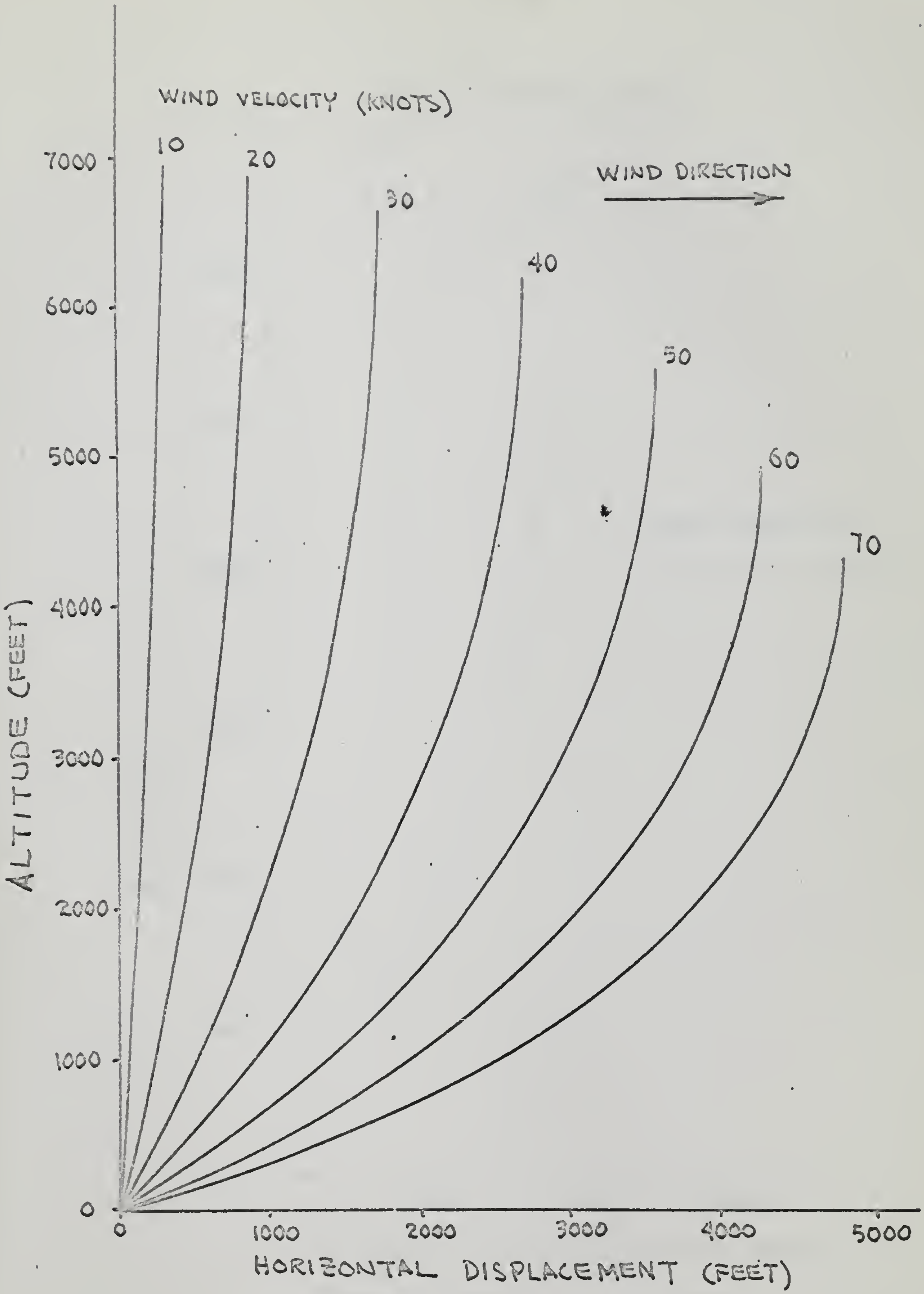
The Critical Angle

| Wind Velocity (knots) | Critical Angle (degrees) |
|---|---|
| 0 | 90 |
| 5 | 89.99 |
| 10 | 89.82 |
| 15 | 89.09 |
| 20 | 87.14 |
| 25 | 83.13 |
| 30 | 76.56 |
| 35 | 66.66 |
| 40 | 56.30 |
| 45 | 46.89 |
| 50 | 39.05 |
| 55 | 32.80 |
| 60 | 27.83 |
| 65 | 23.86 |
| 70 | 20.66 |
| 75 | 18.05 |

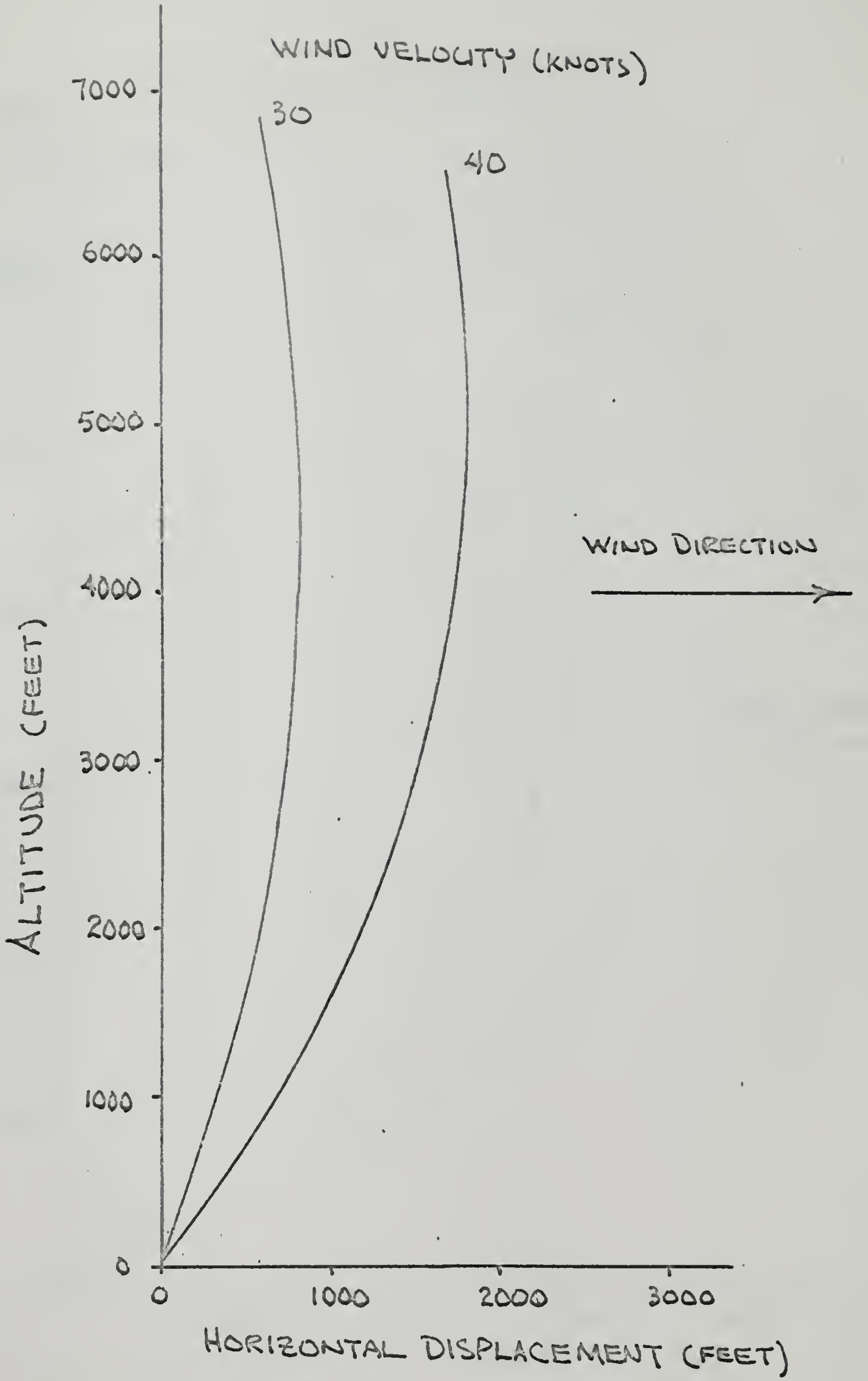
# APPENDIX III

## Antenna Physical Shape

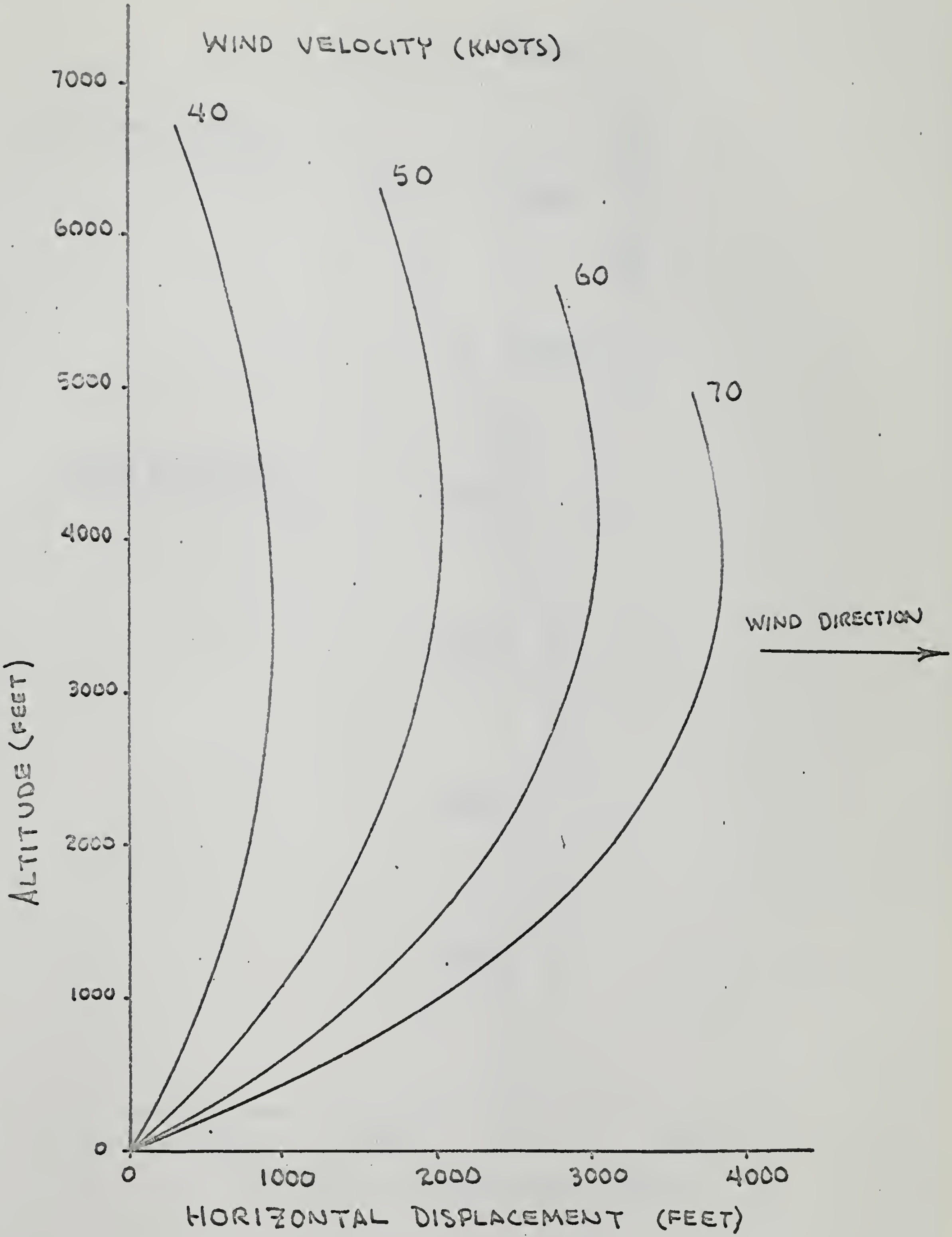
The figures presented in this Appendix show the actual physical shape of an antenna cable 7000 feet in length and with a top tension of 4000 pounds. Curves are shown for values of top angle equal to 90, 80, 70, 60, and 50 degrees and for typical values of wind velocity.



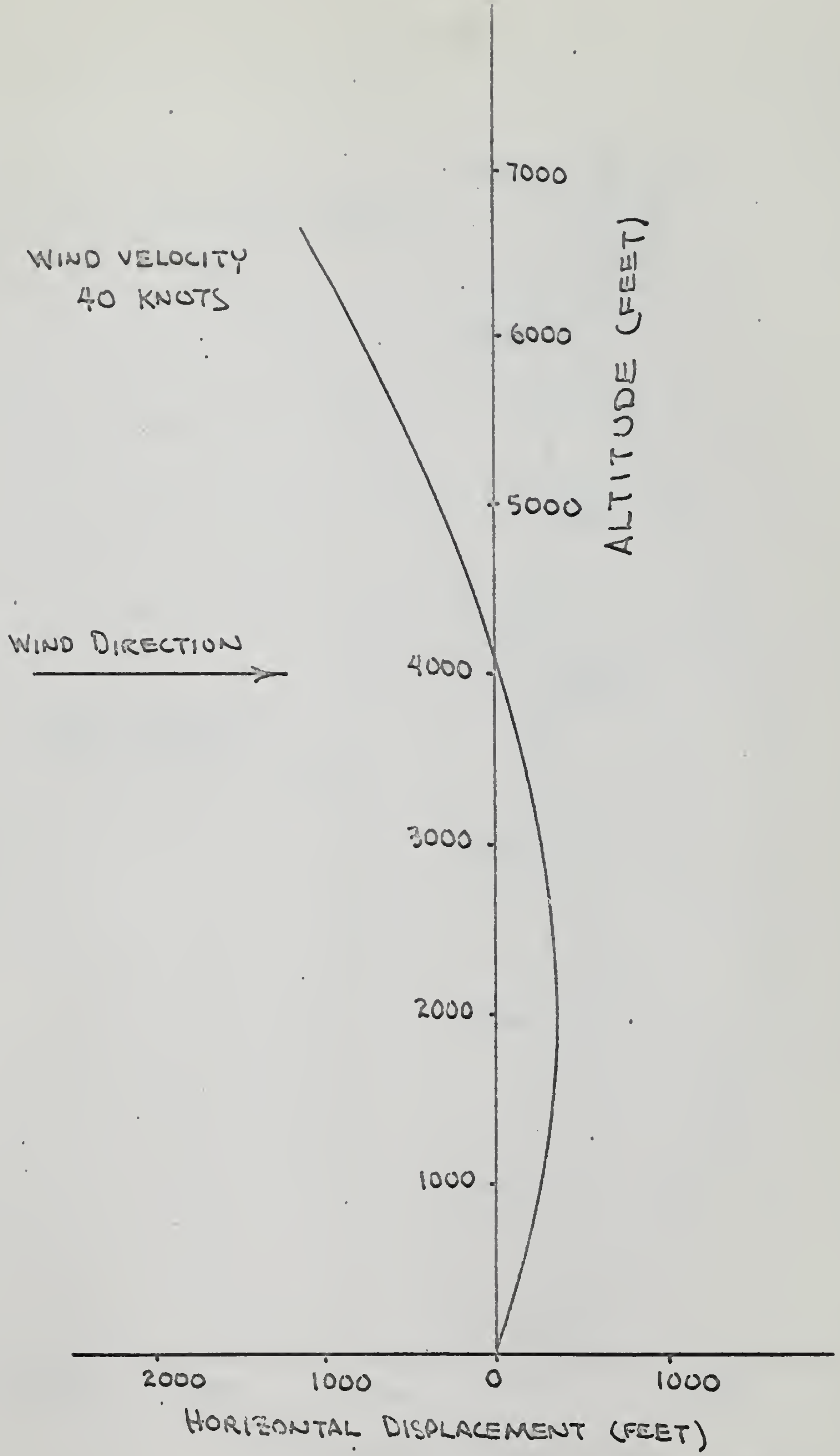

CABLE LENGTH = 7000 FEET
TOP TENSION = 4000 POUNDS
TOP ANGLE = 90°




CABLE LENGTH = 7000 FEET
TOP TENSION = 4000 POUNDS
TOP ANGLE = 80°



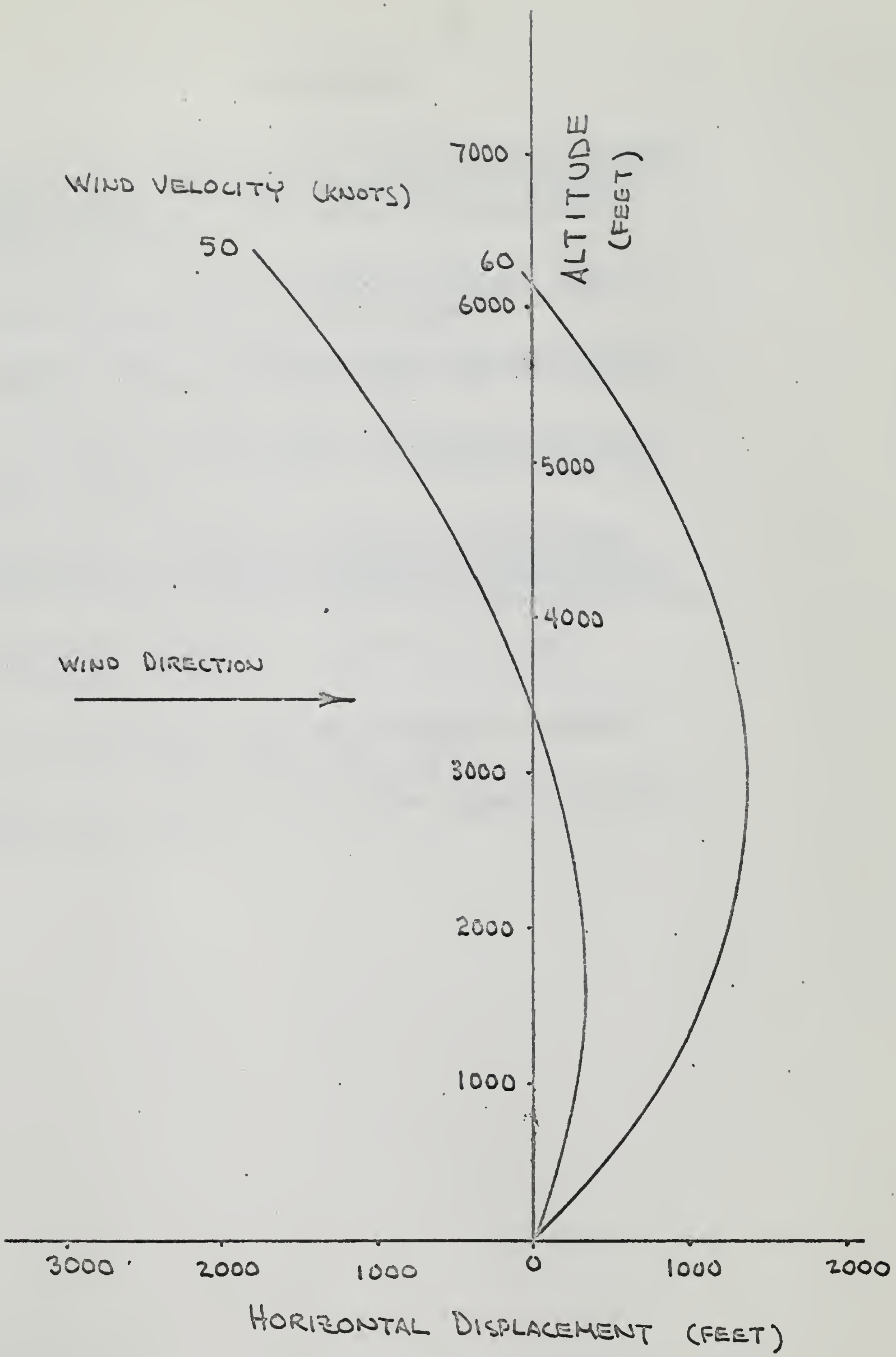

CABLE LENGTH = 7000 FEET
TOP TENSION = 4000 POUNDS
TOP ANGLE = 70°




CABLE LENGTH = 7000 FEET

TOP TENSION = 4000 POUNDS

TOP ANGLE = 60°




CABLE LENGTH = 7000 FEET
TOP TENSION = 4000 POUNDS
TOP ANGLE = 50°

## BIBLIOGRAPHY


1. L. F. Fehlner and E. V. Nice, "EIS CABLE CONFIGURATIONS" General Report Number 102, October 31, 1957, Applied Physics Laboratory, John Hopkins University, Silver Spring, Maryland

2. Henry Jasik, "ANTENNA ENGINEERING HANDBOOK", McGraw-Hill Book Company, INC., New York, 1961

3. Edward C. Jordan, "ELECTROMAGNETIC WAVES AND RADIATING SYSTEMS", Prentice-Hall, INC., 1950

4. B. O. Pierce, "A SHORT TABLE OF INTEGRALS", Ginn and Company, Third Revised Edition, Equations 300 and 303, pages 41 and 42

5. Leonard Pode, "TABLES FOR COMPUTING THE EQUILIBRIUM CONFIGURATION OF A FLEXIBLE CABLE IN A UNIFORM STREAM" David W. Taylor Model Basin Report 687, dated March 1951

6. Curt Protze, German Patent Application T25,821, dated March 14, 1964

7. J. B. Scarborough, "NUMERICAL MATHEMATICAL ANALYSIS", John Hopkins Press, 1930, p.119, p.158

8. S. A. Schelkunoff, "ADVANCED ANTENNA THEORY", John Wiley & Sons, INC., 1952

Thesis 79486
C525 Coates
The effect of blow-
down on the effective
height of a helicopter
supported VLF high-wire
antenna.